Srednicki Chapter 37

QFT Problems & Solutions

A. George

March 1, 2013

Srednicki 37.1. Verify that equations 37.13 and 37.14 follow from equations 37.4 and 37.5.

It is perhaps more instructive to consider equation 37.14 first.

The seems baffling because these matrices don't even have the same dimension. How then can we get a single anti-commutation relation? The key is to remember that this is written in index notation. We have:

$$\{\Psi_\alpha(x,t), \overline{\Psi}_\beta(y,t)\}$$

Since this is index notation, we simply allow α and β to run over 0 or 1. Then:

$$\{\Psi_\alpha(x,t), \overline{\Psi}_\beta(y,t)\} = \begin{pmatrix} \{\chi_c(x), \xi^a(y)\} & \{\chi_c(x), \chi^\dagger_{\dot{a}}(y)\} \\ \{\xi^{\dagger\dot{c}}(x), \xi^a(y)\} & \{\xi^{\dagger\dot{c}}(x), \chi^\dagger_{\dot{a}}(y)\} \end{pmatrix}$$

Two of these immediately vanish, as they are totally separate fields which of course anti-commute.

$$\{\Psi_\alpha(x,t), \overline{\Psi}_\beta(y,t)\} = \begin{pmatrix} 0 & \{\chi_c(x), \chi^\dagger_{\dot{a}}(y)\} \\ \{\xi^{\dagger\dot{c}}(x), \xi^a(y)\} & 0 \end{pmatrix} \tag{37.1.1}$$

Now equation 37.5 gives:

$$\{\psi_a(x), \pi^c(y)\} = i\delta_a{}^c\delta^3(x-y)$$

Using the time derivative of equation 37.1, we have:

$$\{\psi_a(x), i\psi^\dagger_{\dot{d}}\overline{\sigma}^{0\dot{d}c}\} = i\delta_a{}^c\delta^3(x-y)$$

We can drop the i terms. Further, the $\overline{\sigma}^0 = I$ will commute with everything. Then:

$$\{\psi_a(x), \psi^\dagger_{\dot{d}}\}\overline{\sigma}^{0\dot{d}c} = \delta_a{}^c\delta^3(x-y)$$

We multiply both sides by $\sigma^0_{c\dot{e}}$. Then:

$$\{\psi_a(x), \psi^\dagger_{\dot{d}}\}\delta^{\dot{d}}_{\ \dot{e}} = \delta_a{}^c\sigma^0{}_{c\dot{e}}\delta^3(x-y)$$

which gives:

$$\{\psi_a(x), \psi^\dagger_{\dot{e}}\} = \sigma^0_{a\dot{e}}\delta^3(x-y)$$

Lowering the indices, we have:

$$\{\psi_a(x), \psi^\dagger_{\dot{e}}\} = \sigma^0_{a\dot{e}}\delta^3(x-y) \tag{37.1.2}$$

Using this in equation (37.1.1), we have:

$$\{\Psi_\alpha(x,t), \overline{\Psi}_\beta(y,t)\} = \begin{pmatrix} 0 & \sigma^0_{c\dot{a}}\delta^3(x-y) \\ \{\xi^{\dagger\dot{c}}(x), \xi^a(y)\} & 0 \end{pmatrix}$$

Raising the indices in (37.1.2), we can solve for the remaining anti-commutator:

$$\{\Psi_\alpha(x,t), \overline{\Psi}_\beta(y,t)\} = \begin{pmatrix} 0 & \sigma^0_{c\dot{a}}\delta^3(x-y) \\ \overline{\sigma}^{0\dot{a}c}\delta^3(x-y) & 0 \end{pmatrix}$$

which gives, using equation 37.15:

$$\{\Psi_\alpha(x,t), \overline{\Psi}_\beta(y,t)\} = \gamma^0_{\alpha\beta}\delta^3(x-y)$$

□

Now for 37.13. We begin in the same way as before:

$$\{\Psi_\alpha(x,t), \Psi_\beta(y,t)\} = \begin{pmatrix} \{\chi_c, \chi_a\} & \{\chi_c, \xi^{\dagger\dot{a}}\} \\ \{\xi^{\dagger\dot{c}}, \chi_a\} & \{\xi^{\dagger\dot{c}}, \xi^{\dagger\dot{a}}\} \end{pmatrix}$$

Now χ and ξ are two totally different fields, and all components of these fields will anti-commute. Then the off-diagonal terms vanish. The diagonal terms of course vanish by 37.4. Then:

$$\{\Psi_\alpha(x,t), \Psi_\beta(y,t)\} = 0$$

which is equation 37.13. □