Srednicki Chapter 33

QFT Problems & Solutions

A. George

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Srednicki 33.1. Express $A^{\mu\nu}(x)$, $S^{\mu\nu}(x)$, and $T(x)$ in terms of $B^{\mu\nu}(x)$.

We have:

$$B^{\mu\nu} = A^{\mu\nu} + S^{\mu\nu} + \frac{1}{4}g^{\mu\nu}T$$

Start by contracting this with $g_{\mu\nu}$:

$$g_{\mu\nu}B^{\mu\nu} = g_{\mu\nu}A^{\mu\nu} + g_{\mu\nu}S^{\mu\nu} + \frac{1}{4}g_{\mu\nu}g^{\mu\nu}T$$

Manipulating the indices, and recalling that g acts as an index raiser/lowerer while $A^{\mu}_{\ \mu} = Tr(A)$, we have:

$$Tr(B) = Tr(A) + Tr(S) + T$$

It is stated in the text that Tr(S) = 0. Furthermore, an antisymmetric matrix must by definition have a trace of zero. Hence:

$$\boxed{T(x) = B^{\mu}_{\ \mu}(x)}$$

Next we'll go back to the definition of B:

$$B^{\mu\nu} + B^{\nu\mu} = A^{\mu\nu} + A^{\nu\mu} + S^{\mu\nu} + S^{\nu\mu} + \frac{1}{4}(g^{\mu\nu} + g^{\nu\mu})T$$

Now we reverse the order of the indices on the right:

$$B^{\mu\nu} + B^{\nu\mu} = A^{\mu\nu} - A^{\mu\nu} + S^{\mu\nu} + S^{\mu\nu} + \frac{1}{4}(g^{\mu\nu} + g^{\mu\nu})T$$

This gives:

$$B^{\mu\nu} + B^{\nu\mu} = 2S^{\mu\nu} + \frac{1}{2}g^{\mu\nu}T$$

Using the definition of T:

$$S^{\mu\nu}(x) = \frac{1}{2}\left(B^{\mu\nu}(x) + B^{\nu\mu}\right) - \frac{1}{4}g^{\mu\nu}B^{\mu}_{\ \mu}$$

Hence:

$$\boxed{S^{\mu\nu}(x) = \frac{B^{\mu\nu}(x) + B^{\nu\mu}(x)}{2} - \frac{B^{\mu\nu}(x)}{4}}$$

Going back again to the definition of B:

$$B^{\mu\nu} - B^{\nu\mu} = A^{\mu\nu} - A^{\nu\mu} + S^{\mu\nu} - S^{\nu\mu} + \frac{1}{4}(g^{\mu\nu} - g^{\nu\mu})T$$

This gives:

$$\boxed{A^{\mu\nu}(x) = \frac{B^{\mu\nu}(x) - B^{\nu\mu}(x)}{2}}$$

Srednicki 33.2. Verify that equations 33.18-33.20 follow from equations 33.11-33.13.

This is trivial. Use equation 33.16:

$$[N_i, N_j] = \frac{1}{4}\left[J_i - iK_i, J_j - iK_j\right]$$

and then 33.11-33.13

$$[N_i, N_j] = \frac{i}{2}\varepsilon_{ijk}\left[J_k - iK_k\right]$$

and finally 33.16 again:

$$\boxed{[N_i, N_j] = \frac{i}{2}\varepsilon_{ijk}N_k}$$

Taking the Hermitian conjugate, we obtain 33.19:

$$\boxed{[N_i^\dagger, N_j^\dagger] = i\varepsilon_{ijk}N_k^\dagger}$$

And finally we use 33.16, 33.17 to write:

$$[N_i, N_j^\dagger] = \frac{1}{4}\left[J_i - iK_i, J_j + iK_i\right]$$

Using equations 33.11-33.13, we see that:

$$\boxed{[N_i, N_j^\dagger] = 0}$$