Srednicki Chapter 27

QFT Problems & Solutions

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Srednicki 27.1. Suppose that we have a theory with (neglecting higher order corrections)

$$\boldsymbol{\beta(\alpha) = b_1\alpha^2}$$

$$\boldsymbol{\gamma_m(\alpha) = c_1\alpha}$$

Show that

$$\boldsymbol{m(\mu_2) = \left[\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right]^{c_1/b_1} m(\mu_1)}$$

Using the definition of the beta function:

$$\frac{1}{m}\frac{dm}{d\ln\mu} = c_1\alpha$$

and the definition of the anomolous mass dimension:

$$\frac{d\alpha}{d\ln\mu} = b_1\alpha^2$$

We solve the beta function for $\mathrm{d}\ln\mu$:

$$d\ln\mu = \frac{dm}{mc_1\alpha}$$

Plugging this into the anomolous mass dimension:

$$\frac{d\alpha}{dm}mc_1\alpha = b_1\alpha^2$$

which gives:

$$\frac{1}{\alpha}d\alpha = \frac{b_1}{c_1}\frac{1}{m}dm$$

Integrating:

$$\int_{\mu_1}^{\mu_2}\frac{1}{\alpha}d\alpha = \int_{\mu_1}^{\mu_2}\frac{b_1}{c_1}\frac{1}{m}dm$$

which gives:

$$\ln\left[\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right] = \frac{b_1}{c_1}\ln\left[\frac{m(\mu_2)}{m(\mu_1)}\right]$$

Using properties of logs:

$$\ln\left[\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right] = \ln\left[\left(\frac{m(\mu_2)}{m(\mu_1)}\right)^{b_1/c_1}\right]$$

Taking the exponential of both sides:

$$\frac{\alpha(\mu_2)}{\alpha(\mu_1)} = \left(\frac{m(\mu_2)}{m(\mu_1)}\right)^{b_1/c_1}$$

Moving the powers:

$$\left(\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right)^{c_1/b_1} = \frac{m(\mu_2)}{m(\mu_1)}$$

which gives:

$$m(\mu_2) = \left(\frac{\alpha(\mu_2)}{\alpha(\mu_1)}\right)^{c_1/b_1} m(\mu_1)$$

as expected.