Srednicki Chapter 18

QFT Problems & Solutions

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Srednicki 18.1. In any number d of spacetime dimensions, a *Dirac field* $\Psi_\alpha(x)$ carries a spin index α, and has a kinetic term of the form $i\bar{\Psi}\gamma^\mu\partial_\mu\Psi$, where we have suppressed the spin indices; the *gamma matrices* γ^μ are dimensionless.

(a) What is the mass dimension $[\Psi]$ of the field Ψ?

Equation 12.9 tells us that the $[\mathcal{L}] = d$, and 12.3 tells us that $[\partial^\mu] = 1$. So the fields must have a combined mass dimension d-1, $\frac{d-1}{2}$ each.

(b) Consider interactions of the form $g_n(\bar{\Psi}\Psi)^n$, where $n \geq 2$ is an integer. What is the mass dimension $[g_n]$ of g_n?

$$[\mathcal{L}] = [g][\bar{\Psi}\Psi]^n$$

Using the result of part (a),

$$d = [g] + (d-1)n$$

which implies:

$$[g] = d - n(d-1)$$

(c) Consider interactions of the form $g_{m,n}\phi^m(\bar{\Psi}\Psi)^n$, where ϕ is a scalar field, and $m \geq 1$ and $n \geq 1$ are integers. What is the mass dimension $[g_{m,n}]$ of $g_{m,n}$?

As before, we have:

$$d = [g_{m,n}] + m[\phi] + n(d-1)$$

Using equation 12.10:

$$[g_{m,n}] = d - \frac{m}{2}(d-2) - n(d-1)$$

(d) In d = 4 spacetime dimensions, which of these interactions are allowed in a renormalizable theory?

We need the coupling constant to be nonnegative. In four dimensions, the theory of part b has $[g] = 4 - 3n$, so the theory is renormalizable only if $n = 1$. This is not a interaction term, so this is no good. The theory of part (c), on the other hand, has $[g] = 4 - m - 3n$. If $m = n = 1$, then we have a reasonable vertex that will not diverge.