Srednicki Chapter 17

QFT Problems & Solutions

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Srednicki 17.1. Verify equation 17.3

We start by equating the right hand sides of the last two equations on page 115:

$$x_1(\ell - k_1)^2 + x_2(\ell + k_2)^2 + x_3(\ell + k_2 + k_3)^2 + x_4\ell^2 + m^2 = q^2 + D_{1234}$$

Expanding this out:

$$x_1(\ell^2 - 2\ell k_1 + k_1^2) + x_2(\ell^2 + 2\ell k_2 + k_2^2) + x_3(\ell^2 + k_2^2 + k_3^2 + 2\ell k_2 + 2\ell k_3 + 2k_2k_3) + x_4\ell^2 + m^2 = q^2 + D_{1234}$$

We know that $x_1 + x_3 + x_3 + x_4 = 1$, so:

$$\ell^2 + x_1(-2\ell k_1 + k_1^2) + x_2(2\ell k_2 + k_2^2) + x_3(k_2^2 + k_3^2 + 2\ell k_2 + 2\ell k_3 + 2k_2k_3) + m^2 = q^2 + D_{1234}$$

Expanding out the remaining terms:

$$\ell^2 - 2\ell k_1x_1 + k_1^2x_1 + 2\ell k_2x_2 + k_2^2x_2 + x_3k_2^2 + x_3k_3^2 + 2\ell k_2x_3 + 2\ell k_3x_3 + 2k_2k_3x_3 + m^2 = q^2 + D_{1234}$$

This is just a verification, not a proof, so it's OK to manipulate the right hand side as well (so long as we don't make any irreversible steps). Thus:

$$\ell^2 - 2\ell k_1x_1 + k_1^2x_1 + 2\ell k_2x_2 + k_2^2x_2 + x_3k_2^2 + x_3k_3^2 + 2\ell k_2x_3 + 2\ell k_3x_3 + 2k_2k_3x_3 + m^2$$

$$= (\ell - x_1k_1 + x_2k_2 + x_3k_2 + x_3k_3)^2 + D_{1234}$$

which gives:

$$\ell^2 - 2\ell k_1x_1 + k_1^2x_1 + 2\ell k_2x_2 + k_2^2x_2 + x_3k_2^2 + x_3k_3^2 + 2\ell k_2x_3 + 2\ell k_3x_3 + 2k_2k_3x_3 + m^2$$

$$= \ell^2 + x_1^2k_1^2 + x_2^2k_2^2 + x_3^2k_2^2 + x_3^2k_3^2 - 2\ell x_1k_1 + 2\ell x_2k_2 + 2\ell x_3k_2 + 2\ell x_3k_3 - 2x_1k_1x_2k_2 - 2x_1k_1x_3k_2$$
$$-2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

Cancelling some terms:

$$k_1^2x_1 + k_2^2x_2 + x_3k_2^2 + x_3k_3^2 + 2k_2k_3x_3 + m^2 = x_1^2k_1^2 + x_2^2k_2^2 + x_3^2k_2^2 + x_3^2k_3^2 - 2x_1k_1x_2k_2 - 2x_1k_1x_3k_2$$

$$-2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

On this first term on the right hand side, I will invoke $x_1 = 1 - x_2 - x_3 - x_4$. Then,

$$k_1^2x_1+k_2^2x_2+x_3k_2^2+x_3k_3^2+2k_2k_3x_3+m^2 = x_1(1-x_2-x_3-x_4)k_1^2+x_2^2k_2^2+x_3^2k_2^2+x_3^2k_3^2-2x_1k_1x_2k_2-2x_1k_1x_3k_2$$

$$-2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

which gives:

$$k_1^2x_1+k_2^2x_2+x_3k_2^2+x_3k_3^2+2k_2k_3x_3+m^2 = x_1k_1^2-x_1x_2k_1^2-x_1x_3k_1^2-x_1x_4k_1^2+x_2^2k_2^2+x_3^2k_2^2+x_3^2k_3^2-2x_1k_1x_2k_2$$

$$-2x_1k_1x_3k_2 - 2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

Cancelling and moving some terms:

$$x_1x_4k_1^2+x_1x_2k_1^2+x_1x_3k_1^2+k_2^2x_2+x_3k_2^2+x_3k_3^2+2k_2k_3x_3+m^2 = x_2^2k_2^2+x_3^2k_2^2+x_3^2k_3^2-2x_1k_1x_2k_2-2x_1k_1x_3k_2$$

$$-2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

On this first term on the right hand side, I will invoke $x_2 = 1 - x_1 - x_3 - x_4$. Then,

$$x_1x_4k_1^2+x_1x_2k_1^2+x_1x_3k_1^2+k_2^2x_2+x_3k_2^2+x_3k_3^2+2k_2k_3x_3+m^2 = x_2k_2^2-x_1x_2k_2^2-x_3x_2k_2^2-x_4x_2k_2^2+x_3^2k_2^2$$

$$+x_3^2k_3^2 - 2x_1k_1x_2k_2 - 2x_1k_1x_3k_2 - 2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

Cancelling and moving some terms:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_1x_2k_1^2+x_1x_3k_1^2+x_3k_2^2+x_3k_3^2+2k_2k_3x_3+m^2+x_1x_2k_2^2+x_3x_2k_2^2 = x_3^2k_2^2+x_3^2k_3^2$$

$$-2x_1k_1x_2k_2 - 2x_1k_1x_3k_2 - 2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

On this first term on the right hand side, I will invoke $x_3 = 1 - x_1 - x_2 - x_4$. Then,

$$x_1x_4k_1^2+x_2x_4k_2^2+x_1x_2k_1^2+x_1x_3k_1^2+x_3k_2^2+x_3k_3^2+2k_2k_3x_3+m^2+x_1x_2k_2^2+x_3x_2k_2^2 = (1-x_1-x_2-x_4)x_3k_2^2+x_3^2k_3^2$$

$$-2x_1k_1x_2k_2 - 2x_1k_1x_3k_2 - 2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

Cancelling and moving some terms:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_1x_2(k_1+k_2)^2+x_1x_3k_1^2+x_3k_3^2+2k_2k_3x_3+m^2+x_3x_2k_2^2+x_2x_3k_2^2+x_1x_3k_2^2+x_4x_3k_2^2 =$$

$$+x_3^2k_3^2 - 2x_1k_1x_3k_2 - 2x_1k_1x_3k_3 + 2x_2k_2x_3k_2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

On this first term on the right hand side, I will invoke $x_3 = 1 - x_1 - x_2 - x_4$. Then,

$$x_1x_4k_1^2+x_2x_4k_2^2+x_1x_2(k_1+k_2)^2+x_1x_3k_1^2+x_3k_3^2+2k_2k_3x_3+m^2+x_3x_2k_2^2+x_2x_3k_2^2+x_1x_3k_2^2+x_4x_3k_2^2 =$$

$$+x_3k_3^2-x_1x_3k_3^2-x_2x_3k_3^2-x_4x_3k_3^2-2x_1k_1x_3k_2-2x_1k_1x_3k_3+2x_2k_2x_3k_2+2x_2k_2x_3k_3+2x_3^2k_2k_3+D_{1234}$$

Cancelling and moving some terms:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_2x_3k_3^2+x_1x_2(k_1+k_2)^2+x_3x_4k_2^2+x_3x_4k_3^2+x_1x_3(k_1^2+k_2^2+k_3^2+2k_1k_2+2k_1k_3)$$

$$+x_3k_3^2 + 2k_2k_3x_3 + m^2 + x_3x_2k_2^2 + x_2x_3k_2^2 = x_3k_3^2 + 2x_2x_3k_2^2 + 2x_2k_2x_3k_3 + 2x_3^2k_2k_3 + D_{1234}$$

Notice that $k_1+k_2+k_3=-k_4 \implies (k_1+k_2+k_3)^2=k_4^2 \implies k_1^2+k_2^2+k_3^2+2k_1k_2+2k_1k_3+2k_2k_3=k_4^2$. So:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_2x_3k_3^2+x_1x_2(k_1+k_2)^2+x_3x_4k_2^2+x_3x_4k_3^2+x_1x_3k_4^2-2x_1x_3k_2k_3$$

$$+x_3k_3^2+2k_2k_3x_3+m^2+x_3x_2k_2^2+x_2x_3k_2^2=x_3k_3^2+2x_2x_3k_2^2+2x_2k_2x_3k_3+2x_3^2k_2k_3+D_{1234}$$

Moving some terms:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_2x_3k_3^2+x_1x_3k_4^2+x_1x_2(k_1+k_2)^2+x_3x_4k_2^2+x_3x_4k_3^2-2x_1x_3k_2k_3$$

$$+x_3k_3^2+2k_2k_3x_3+m^2+x_3x_2k_2^2+x_2x_3k_2^2=x_3k_3^2+2x_2x_3k_2^2+2x_2k_2x_3k_3+2x_3^2k_2k_3+D_{1234}$$

Now I want to add 0, in the form $2x_3x_4k_2k_3-2x_3x_4k_2k_3$. This allows us to write:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_2x_3k_3^2+x_1x_3k_4^2+x_1x_2(k_1+k_2)^2+x_3x_4k_2^2+x_3x_4k_2k_3+x_3x_4k_3^2-2x_3x_4k_2k_3-2x_1x_3k_2k_3$$

$$+x_3k_3^2+2k_2k_3x_3+m^2+x_3x_2k_2^2+x_2x_3k_2^2=x_3k_3^2+2x_2x_3k_2^2+2x_2k_2x_3k_3+2x_3^2k_2k_3+D_{1234}$$

Grouping some terms:

$$x_1x_4k_1^2+x_2x_4k_2^2+x_2x_3k_3^2+x_1x_3k_4^2+x_1x_2(k_1+k_2)^2+x_3x_4(k_2+k_3)^2+m^2-2x_3x_4k_2k_3-2x_1x_3k_2k_3$$

$$+x_3k_3^2+2k_2k_3x_3+x_3x_2k_2^2+x_2x_3k_2^2=x_3k_3^2+2x_2x_3k_2^2+2x_2k_2x_3k_3+2x_3^2k_2k_3+D_{1234}$$

Again, this is a verification rather than a proof, so we can use equation 17.3, and cancel D_{1234} with the first seven terms:

$$-2x_3x_4k_2k_3-2x_1x_3k_2k_3+x_3k_3^2+2k_2k_3x_3+x_3x_2k_2^2+x_2x_3k_2^2=x_3k_3^2+2x_2x_3k_2^2+2x_2k_2x_3k_3+2x_3^2k_2k_3$$

Cancelling and moving some terms:

$$x_2k_2x_3k_3+x_3^2k_2k_3+x_3x_4k_2k_3+x_1x_3k_2k_3-k_2k_3x_3=0$$

On the second term, I will invoke $x_3=1-x_1-x_2-x_4$. Then,

$$x_2k_2x_3k_3+(1-x_1-x_2-x_4)x_3k_2k_3+x_3x_4k_2k_3+x_1x_3k_2k_3-k_2k_3x_3=0$$

Simplifying:

$$x_2k_2x_3k_3+x_3k_2k_3-x_1x_3k_2k_3-x_2x_3k_2k_3-x_3x_4k_2k_3+x_3x_4k_2k_3+x_1x_3k_2k_3-k_2k_3x_3=0$$

We cancel the second term with the eighth, the first with the fourth, the third with the seventh, and the fourth with the fifth, to find that:

$$0=0$$

which is obviously true, completing our verification.

Note: Had this been a proof rather than a verification, no additional math would be required. We would have had to drag around certain terms until the end, making the notation much more cumbersome, however.

Note 2: I'm not sure what the point of this problem was. Algebra practice? The logical problem for this section would have been to derive V_5, but that would have required much more algebra even than this problem.