QFT

Chapter 71: The Path Integral for Non-Interacting Gauge Theory

Representation Theory

- Here we evaluate the path integral for non-Abelian gauge theory
  - We know how to do this (Fourier transform, define new variables, complete the square, define the propagator, etc.)
  - The problem is the gauge invariance: we have an extra degree of freedom
    - In Abelian gauge theory (section 57), we resolved this difficulty by arguing that the perpendicular component of the polarization did not contribute, and could be dropped
    - In non-Abelian gauge theory, we do not have this luxury.
  - Srednicki presents an argument that gets us to where we can use Feynman Rules. I won't go through the entire thing again, but I will hit the high points.

Ghost Fields

- The basic idea is to insert a delta function that imposes the gauge condition.
- To calculate the functional determinant, we have to introduce two Grassmann fields and a Grassmann action.
  - We call these fields “ghosts”, because the associated particles do not exist, and the production amplitude is therefore zero
  - But, they do affect scattering amplitudes
  - In other words, adding these additional fields is necessary just to break the gauge invariance.

- Why don’t we have ghost fields in Abelian gauge theories?
  - There’s no interaction term for the ghost field because the structure constants are zero. So it’s just an extra free fields, which we integrate out.

- So are these particles, or not?
  - No. They’re just mathematical devices we use to get the math to work out.
  - If they did exist, they would be anti-commuting spin-0 particles, which would violate the spin-statistics theorem
  - They look like real particles in Feynman Diagrams, but they’re actually quasiparticles.

Gauge Fixing Term

- We will have a free parameter in our path integral (Srednicki names it ω).
  - Since it's free, it can't hurt anything to redefine it and then integrate over it. This can only affect the normalization.
  - We take advantage of this freedom to introduce a gauge-fixing term to the Lagrangian, written in the R-xi form (remember, this is the gauge condition where you can choose xi to be whatever you want).