QFT

Chapter 69: Nonabelian Gauge Theory

Overview

- We've talked about non-Abelian scalar theories. Recall:
  - We can require a symmetry to be local and not global from the beginning; the result is basis vectors that do not necessarily commute (hence, "non-Abelian theory").
    - This is what we do for QCD
    - But shouldn't QED by local as well? So far our theory of QED is consistent with locality, but does not require it. We'll show later that we can require locality by insisting on massless photon that does not self-interact.
  - In any space, we can find a set of generator matrices that span the space. In non-Abelian theories, these will not commute, but will instead obey

$$[T^a, T^b] = i f^{abc} T^c$$

  - where f^{abc} is the structure constant, and is a real number.
  - There are also infinitesimal transformations defined in terms of θ
  - We normally choose the generator matrices to obey a particular normalization.

Gauge Transformations

- We define in SU(N) the gauge field as a traceless NxN matrix of fields.
  - The U in SU(N) means unitary ie determinant 1. Since ln det A = Tr ln A, the basis matrices must be traceless. This removes one degree of freedom, so the basis has $N^2 - 1$ members.
    - Physically, we will eventulaly interpret these generator matrices as quarks, so this is why there are 8 quarks.
  - We propose the following gauge transformation property (we show in problem 69.2 that this is actually representation-independent).

$$A_\mu(x) \to U(x) A_\mu(x) U^\dagger(x) + \frac{i}{g} U(x) \partial_\mu U^\dagger(x)$$

Field Strength

- We use the usual definition for field strength

$$F_{\mu\nu}(x) = \frac{i}{g}[D_\mu, D_\nu]$$

- but this time, things don't vanish as nicely; this could therefore serve as a kinetic term for a gauge field.

$$\mathcal{L} = -\frac{1}{4}F^{c\mu\nu}F^c_{\mu\nu}$$

- We show in fact that:

$$F^c_{\mu\nu} = \partial_\mu A^c_\nu - \partial_\nu A^c_\mu + g f^{abc} A^a_\mu A^b_\nu$$

- Thus, a *local* gauge symmetry (requiring gauge invariance at every point in space, not just globally) requires from the beginning that the terms interact with each other.
  - Or we could try to find a different kinetic term, but we need something that depends on derivatives and is gauge-invariant, so our options are few.

QCD

- Now let's consider QCD
  - SU(3) group, for three colors
  - Six flavors, one for each quark
  - Try using the field strength for our kinetic term.
  - For our term with no derivatives, and our term with one derivative, we only have one option each.
  - Thus we have:

$$\mathcal{L} = i\overline{\Psi}_{iI}\ \not{D}_i j \Psi_{jI} - m_I \overline{\Psi}_{iI} \Psi_{iI} - \frac{1}{2} \mathrm{Tr}(F^{\mu\nu} F_{\mu n u})$$

  - For simplicity, we have omitted electric charge of the quarks
  - Our course, we sum over all indices in order to include all the quarks.
  - The generator matrices (used in the derivative) are defined of terms of the T matrices, of which there are 8. These correspond to the gluons.

General Non-Abelian Gauge Theory

- Most of our results here extent to general groups, regardless of representation.
- To get general results, we work in terms of T_R, the generator matrices in any given space
  - We'll talk more about group representations in the next chapter.

- By the way, non-Abelian Gauge Theory (any theory with f^{abc} nonzero) is also called Yang-Mills Theory.