QFT

Chapter 67: Ward Identities in Quantum Electrodynamics I
Overview

• Here we derive the Ward Identity, which we state here.

Given a scattering amplitude:

\[ \mathcal{I} = \varepsilon^\mu M_\mu \]

We have:

\[ k^\mu M_\mu = 0 \]

• Physically, this means (among other things) that the photon polarization must be transverse only.
  • Any longitudinal component will be in the direction of \( k \) and will therefore not contribute to the scattering amplitude.
Proof

• The proof is given in Srednicki, it is not necessary to rehash it here. Let’s hit the high points, however:

• The wave operators in the LSZ function (the K-G equations for scalars; Dirac for fermions) will kill any terms with no singularities.

• In particular, these kill contact terms: terms inside a correlation function whose spacetime argument and indices do not match up with those of any other term.
The Ward-Takahashi Identity

- The Ward-Takahashi identity is the more general case, in which we have
  \[ \partial^\mu \langle 0 | T j_\mu (x) \ldots | 0 \rangle = \text{contact terms} \]

- These contact terms don’t contribute to the transition amplitude, so we specialize to the Ward identity, which was stated earlier.

- Hence, if we replace a photon’s polarization vector with its four-momentum, the transition amplitude will be zero, meaning photons don’t interact.
  - This again illustrates that polarization vectors must be transverse.