QFT

Chapter 67: Ward Identities in Quantum Electrodynamics I

Overview

- Here we derive the Ward Identity, which we state here.

  Given a scattering amplitude:

$$\mathcal{T} = \varepsilon^{\mu}\mathcal{M}_{\mu}$$

  We have:

$$k^{\mu}\mathcal{M}_{\mu} = 0$$

- Physically, this means (among other things) that the photon polarization must be transverse only.
  - Any longitudinal component will be in the direction of k and will therefore not contribute to the scattering amplitude.

Proof

- The proof is given in Srednicki, it is not necessary to rehash it here. Let’s hit the high points, however:

- The wave operators in the LSZ function (the K-G equations for scalars; Dirac for fermions) will kill any terms with no singularities.

- In particular, these kill *contact terms*: terms inside a correlation function whose spacetime argument and indices do not match up with those of any other term.

The Ward-Takahashi Identity

- The Ward-Takahashi identity is the more general case, in which we have

$$\partial^\mu \langle 0|T j_\mu(x) \ldots |0\rangle = \text{contact terms}$$

- These contact terms don't contribute to the transition amplitude, so we specialize to the Ward identity, which was stated earlier.

- Hence, if we replace a photon's polarization vector with its four-momentum, the transition amplitude will be zero, meaning photons don't interact.
  - This again illustrates that polarization vectors must be transverse.