QFT

Chapter 59: Scattering in Spinor Electrodynamics

Overview

- We've done this three or four times already. We use the Feynman rules to determine T, then sum and average, use the completeness relations, simplify, and introduce Mandelstam variables in order to write the squared matrix element for a cross-section.
  - We can then use this in the formulas of chapter 11 to determine the cross-section, which we measure
  - We go over this in detail in the problems.
  - Again, these problems are extremely tedious. However, they are not examples: they are the actual calculations that yield the correct matrix elements (at tree level, ie to ~99.8% accuracy) for observable processes.
    - And this time, these are not obscure processes: Compton scattering for example happens all the time.

One Complication

- The one complication is when have to use the completeness relation for a photon-photon state. There is no completeness relation. What to do?

$$\sum_{\lambda=\pm} \varepsilon_\lambda^{i*}(\vec{k})\varepsilon_\lambda^{j}(\vec{k})$$

- In chapter 55, we required ε, ε*, and k to form an orthonormal basis. In chapter 56, we extended this to four dimensions, which gives:

$$\sum_{\lambda=\pm} \varepsilon_\lambda^{\mu*}(\vec{k})\varepsilon_\lambda^{\nu}(\vec{k}) = g^{\mu\nu} - \hat{t}^\mu \hat{t}^\nu - \hat{z}^\mu \hat{z}^\nu$$

- Now we can write

$$\hat{z}^\mu = \frac{k^\mu + (\hat{t}\cdot k)\hat{t}^\mu}{[k^2 + (\hat{t}\cdot k)^2]^{1/2}}$$

Preface to the Ward Identities

- What do we do with the k terms here?

- Let's take a gauge transformation on the original polarization, $\varepsilon(k) \dashrightarrow \varepsilon(k) - i\,\Gamma\, k$.

- By gauge invariance, the scattering amplitude should not depend on this, so these k terms should drop.
  - Physically, this means that the polarization cannot have any longitudinal component.
  - This is called the Ward identity, which we'll prove later.

- For the moment, we use this to drop the k terms.

The Resolution

- Now we're down to:

$$\hat{z}^{\mu} = \frac{(\hat{t} \cdot k)\hat{t}^{\mu}}{[k^2 + (\hat{t} \cdot k)^2]^{1/2}}$$

- But for photons, $k^2 = 0$, and so we have:

$$\hat{z}^{\mu} = \hat{t}^{\mu}$$

- Hence:

$$\sum_{\lambda=\pm} \varepsilon_{\lambda}^{\mu *}(\vec{k})\varepsilon_{\lambda}^{\nu}(\vec{k}) = g^{\mu\nu}$$