QFT

Chapter 54: Maxwell’s Equations

QFT and Maxwell's Equations

- Now we begin part 3 and turn our attention to spin-1 particles.
  - The only elementary spin-1 particle (that we know of) is the photon.
  - Thus, our spin-1 QFT had better be compatible electrodynamics.

- Recall classical E&M, which we can summarize by Maxwell's Equations, written in our (Heaviside-Lorentz) units:

$$\begin{aligned} \nabla \cdot \vec{E} &= \rho \\ \nabla \times \vec{E} &= -\dot{B} \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times B &= \vec{J} + \dot{\vec{E}} \end{aligned}$$

Gauges

- Now we can uniquely determine the fields by the potentials, which are defined by:

$$\vec{E} = -\nabla\phi - \dot{\vec{A}}$$
$$\vec{B} = \nabla \times \vec{A}$$

- These potentials, on the other hand, are not unique:

$$\phi' = \phi + \dot{\Gamma}$$
$$\vec{A}' = \vec{A} - \nabla\Gamma$$

Relativistic Notation

- We make this relativistic:
  - The field strength tensor combines the electric and magnetic fields into one, **Lorentz-independent** tensor
  - We also define a four-vector of charge current: the three-current in the spatial part, and the charge density in the temporal part.

- These two things together can encapsulate everything in Maxwell's equations

$$(g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)A_\nu + J^\mu = 0$$

Lagrangian

- We need an action that results in Maxwell's equations as the equations of motions.
  - Treat the current as an external source
  - The action should be
    - Lorentz invariant
    - Gauge invariant
    - P and T invariant
    - No more than second order in derivatives
      - because we want a renormalizable theory, and the mass dimension of the derivative is +1.

- There is only one candidate:

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J^{\mu}A_{\mu}$$