QFT

Chapter 46: Spin Sums

Example

- Consider e$^-\varphi$ --> e$^-\varphi$. We determined in the previous chapter that:

$$\mathcal{T} = \overline{u}' A u$$

where A is a bunch of junk, see equation 46.3.

- We then have:

$$|\mathcal{T}|^2 = (\overline{u}' A u)(\overline{u} A u')$$

  - We put this in index notation
  - Then we can reorder freely. We choose an order such that the first index and the last index are equal.
  - This is the trace. Thus:

$$|\mathcal{T}|^2 = \mathrm{Tr}\left[(u'\overline{u}')A(u\overline{u})A\right]$$

- Next we use equation 38.28, and we're done.

$$|\mathcal{T}|^2 = \frac{1}{4}\mathrm{Tr}\left[(1 - s'\gamma_5 \not{z}')(-\not{p}' + m)A(1 - s\gamma_5 \not{z})(-\not{p} + m)A\right]$$

  - All that remains is to learn how to take traces of products of gamma matrices – that's next section

Spins

- In practice, it is not so easy to measure particle spins
  - To us, outgoing electrons of either spin are equally good. Since both are possible, we need to sum over the two decay channels (we have double phase space since there are two equally good possibilities).
  - Similarly, incoming electrons of either spin are equally good. A given electron can only have one spin, so we average over the two possibilities.

- Doing this, our result simplifies to:

$$|\mathcal{T}|^2 = \frac{1}{2}\mathrm{Tr}\left[(-\not{p}' + m)A(-\not{p} + m)A\right]$$

Gamma Matrices

- Our next step is to learn how to deal with the trace of multiple gamma matrices.
  - We can have up to two incoming fermions and two outgoing fermions – 4 spinors
  - When squaring the amplitude, this becomes 8 spinors
  - We sum over the spinors using

$$\sum_{s=\pm} u_s(\vec{p})\overline{u}_s(\vec{p}) = - \not{p} + m$$

this gives four gamma matrices inside the trace

  - Hence, the trace of four gamma matrics should be enough.