QFT

Chapter 45: The Feynman Rules for Dirac Fields

Yukawa Theory

- Yukawa Theory involves one scalar and two Dirac Fields:

$$\mathcal{L}_1 = g\phi\overline{\Psi}\Psi$$

- The mass dimension of g is zero (the scalar is 1 and the spinors are 3/2 as discussed before). This is the most interesting case, as it neither blows up nor becomes trivial at high energy.

- L_1 is invariant under U(1) transformation, thus charge is still conserved. We identify:
  - the b-type particle as the electrons
  - the d-type particle as the positron
  - the scalar as the Higgs Boson

Path Integral

- Now we have a path integral which is a function of the scalar sources (J) and the spinor sources (η).

- We defined the free-field path integral previously, but now we need to multiply two factors of the spinor free-field path integral with one factor of the scalar free-field path integral:

$$Z_0(\bar{\eta}, \eta, J) = \exp\left[i \int d^4x d^4y \; \overline{\eta}(x) S(x-y) \eta(y)\right] \exp\left[\frac{i}{2} \int d^4x d^4y \;\; J(x) \Delta(x-y) J(y)\right]$$

- As in chapter 9, we can do the same thing with the interacting path integral.
  - We take the interacting part into its own exponential, and bring it out in front of the free part
  - Recognize that $L_1(\varphi) = L_1[(1/i)\delta/dJ]$, etc.

Feynman Expansion

- This give:

$$Z(\bar{\eta},\eta,J) = \exp\left[ig\int d^4x \left(\frac{1}{i}\frac{\delta}{\delta J(x)}\right)\left(i\frac{\delta}{\delta\eta_\alpha(x)}\right)\left(\frac{1}{i}\frac{\delta}{\delta\overline{\eta}_\alpha(x)}\right)\right] Z_0(\bar{\eta},\eta,J)$$

- Now we have two exponentials. We use perturbation theory to expand both of them
  - Now we can uniquely assign each term to a category depending on the number of derivatives, and number of propagators.
  - Now we represent each term by a diagram, and assess the value of each diagram in order to construct the series.

Rules for Drawing Diagrams

- To keep the diagrams separate, we decide:
  - Scalar propagators get dashed lines
  - Fermion propagators get solid lines
  - Blobs represent sources
    - Recall that sources are set to zero at the end, so if sources are not exactly canceled by derivatives, the diagram can't contribute to any physical process.
  - Arrows point toward the blob (away from the vertex) for anti-fermions, and away from the bob (toward the vertex) for fermions.
  - Yukawa theory specifies the vertex: two fermions with opposite arrows and a scalar, vertex factor is ig.

- Remember to cancel tadpoles, these can be cancelled by shifting the φ field.
  - Tadpoles are diagrams which, when a single line is cut, divide into two separate diagrams, one of which has no sources

Correlation Functions

- Recall that we can evaluate the correlation functions by writing all the connected diagrams and killing off sources corresponding to the function I want to evaluate, ie:

$$\langle 0|T\Psi\overline{\Psi}\phi\phi|0\rangle = \frac{1}{i}\frac{\delta}{\delta\overline{\eta}} \times i\frac{\delta}{\delta\eta} \times \frac{1}{i}\frac{\delta}{\delta J} \times \frac{1}{i}\frac{\delta}{\delta J} \times iW(\overline{\eta},\eta,J)\Big|_{\overline{\eta}=\eta=J=0}$$

  - This is by analogy with the harmonic oscillator case, see 7.15.

- The problem is that the functional derivative with respect to eta bar anticommutes; this results in a negative sign depending on the order in which the derivatives are taken
  - All we care about is the relative sign; the absolute sign will be irrelevant when we calculate a physical result
  - Procedure: draw diagrams with identical left sides. Define the first diagram to be positive. If the fermion lines on the right hand side of subsequent diagrams are an odd permutation of the first diagram, assign a relative minus sign.

Feynman Rules

- We now have quite a bit of stuff:
  - Rules stated here
  - Form of propagator in chapter 42
  - Substitutions in chapter 41 relating creation/annihilation operators to fermionic field operators

- Putting all this together, we arrive at the Feynman Rules, see page 288-289
  - We'll practice using these in the problems, see problem 45.2.