QFT

Chapter 41: LSZ Reduction for spin-1/2 particles

Overview

- The procedure here is exactly the same as it was for spin-zero particles
  - Construct initial and final states using the vacuum, creation operators, and a fourier transforms
    - Should have fairly well-defined momentum and position
  - Go through the same math as in section 5, only use the Dirac equation rather than the Klein-Gordon Equation
  - The result is the LSZ formula
    - has a slightly different form depending on whether b-type or d-type particles are going in or out.
  - As before, we have to test our implicit assumption that free-field creation operators work in an an interacting theory. The result, as in chapter 5, is that it is true given certain conventions.
    - Also as before, have to renormalize to meet these conventions.

- No point going through the math again
  - It's in the book *twice* – here and in section 5
  - All that's really worth doing is noting the new conventions, as we'll have to follow these going forward….

For the Dirac Field, we must require…

$$\langle 0|\Psi(x)|0\rangle = 0$$
$$\langle p, s, +|\Psi(x)|0\rangle = 0$$
$$\langle p, s, -|\Psi(x)|0\rangle = v_s(p)e^{-ipx}$$
$$\langle p, s, +|\overline{\Psi}(x)|0\rangle = \overline{u}_s(p)e^{-ipx}$$
$$\langle p, s, -|\overline{\Psi}(x)|0\rangle = 0$$

The first is because we want the creation operator acting on the vacuum to give an excited state, not any part of the ground state.

The second and fifth are required by charge conservation. **Equation 39.42 tells us that d particles are negatively charged (for Dirac fields)**. According to the mode expansion of Ψ, Ψacting on the vacuum ket will create a |p, s, -> ket, which must be orthogonal to <p, s, +|. Same with the last equation.

The third and fourth happen like this:
$\langle p|\Psi(x)|0\rangle = \langle p|e^{-ipx}\Psi(0)e^{ipx}|0\rangle = e^{-ipx}\langle p|\Psi(0)|0\rangle$. The correlation function here is now not a number, but a number times a spinor, according again to the mode expansion of Ψ (see problem 41.1).