QFT

Chapter 40: Parity, time reversal, and charge conjugation

Parity

- Parity: reversing space.
  - Operator: diag(1, -1, -1, -1)
  - Two parity transformations should have no effect on any observable.
  - However, any observable must consist of an even number of fermion fields
    - [just because Ψ is a spinor, so need $\Psi^+\Psi$ to get a scalar observable]
    - So, parity transformation should have form:

$$P^{-1}\Psi(x)P = D(\mathcal{P})\Psi(\mathcal{P}x) \qquad P^{-1}d_s^\dagger(\vec{p})P = \eta d_s^\dagger(-\vec{p})$$

where $D(P)^2$ has magnitude 1.
  - Should reverse momentum but not spin. Therefore, we have

$$P^{-1}b_s^\dagger(\vec{p})P = \eta b_s^\dagger(-\vec{p})$$

where η is a possible phase factor.
    - For the same reason above, should have $\eta^2 = \pm 1$.
    - We keep the ηs equal since

Parity II

- Taking the Parity transform of the mode expansion of the Fermionic field operator gives: $D(P) = i\beta$
  - How do we interpret the i? Apply the parity operator to an electron-positron state. The resulting parity is opposite to that of its wave function: the pair has an intrinsic parity of -1.
  - This effects the selection rules for fermion pair annihilation (if parity is conserved).

- Further, parity transformations exchange the left-handed Weyl field for a right-handed one.

- Definitions:
  - Scalars even under parity: scalar
  - Scalar odd under parity: pseudoscalar
  - Vector even under parity: vector or polar vector
  - Vector odd under parity: pseudovector or axial vector

Everything else

- We now repeat this exercise for T and C.
  - No point going through the math again, the results are similar

- The CPT Theorem states that the action must be invariant under a combined C, P, and T transformation.
  - This is because the Lagrangian is built out an Hermitian combination of a set of fields and its derivatives, both of which are invariant under CPT as per the results in the chapter.