QFT

Chapter 39: Canonical Quantization of Spinor Fields II

Overview

- Recall that when we tried to write a Lagrangian for our Spinor field, we found that:
  - Our equation of motion was the Dirac Equation
  - Our field was a four-component field, consisting of a left-handed Weyl Spinor and a right-handed Weyl Spinor
- In chapter 37, we solved the Dirac Equation and wrote the mode expansion of the spinor field.
  - The mode expansion involved spinors and operators
  - In chapter 38, we deduced useful properties for these spinors
  - Now we investigate the operators, and calculate some conserved quantities.

Anticommutation of Creation/Annihilation operators for Spinor Fields

- No need to go through the math again, but Srednicki derives the expansion of the operators in terms of the fields, then uses the canonical anticommutation relations of the fields to derive the operators' anticommutation relations.

- We use these to conclude, as expected, that for one spinor field we have two types of particles, b-type and d-type, each with two possible spin states, + and -.

Hamiltonian – Dirac Fields

- We can write the Hamiltonian in terms of the creation/ annihilation operators, then simplify. The result is that:

$$H = \sum_{s=\pm} \int \widetilde{dp}\, \omega \left[b_s^\dagger(p) b_s(p) + d_s^\dagger(p) d_s(p) \right] - 4\mathcal{E}_0 V$$

- We conclude that the ground state must be annihilated by both the d and b operators (this is a vacuum). Physically, we decide that:
  - The b^+ operator creates a b-particle with spin s
  - The d^+ operator creates a d-particle with spin s

Conserved Charge – Dirac Fields

- We calculate this conserved Noether charge in problem 39.1:

$$Q = \sum_{s=\pm} \int \widetilde{dp}\, \left[b_s^\dagger(p) b_s(p) + d_s^\dagger(p) d_s(p) \right] + \text{constant}$$

  - So, we can create/destroy a d-type particle and a b-type particle simultaneously, but we cannot create one without creating the other.

Majorana Particles

- We can repeat this exercise for Majorana Particles, this time requiring that the Majorana condition be satisfied.
  - The Lagrangian has no U(1) symmetry, so there is no conserved charge this time.
  - The Hamiltonian reduces to

$$H = \sum_{s=\pm} \int \widetilde{dp}\ \omega\ b_s^\dagger(p) b_s(p) - 2\mathcal{E}_0 V$$

    - So, we only have one type of particle
    - Still have two spin states

A note on the barring notation

- This gets a little unclear in the problems, so let me summarize what is meant by barring.
  - Spinors: take the Hermitian conjugate and multiply on the right by β.
  - Gamma matrices: take the Hermitian conjugate and multiply on both sides by β
  - Everything else: take the Hermitian conjugate

- Very informally:
  - We tend to think about numbers generalizing into vectors and matrices
  - We also tend to understand that four vectors are a separete animal, which generalize into tensors. We tend to associate these with Hermitian conjugation
  - We now need to consider spinors as an altogether different animal – generalizing into things like the gamma matrices -- and associate them with this “barring.”