QFT

Chapter 38: Spinor Technology

Review

- In quantum mechanics, we write spin-1/2 objects as Weyl spinors
- In QFT, we use the field operators, which consist of two Weyl spinors: the left-handed one and the right-handed one.
- In the previous section, we showed that the field operator can be decomposed into a 4-dimensional spinor u or v, as well as an operator.
  - We'll discuss the operator next time
  - This time, we discuss the spinors
- The field operator must follow the Dirac equation. Last time, we showed that it follows that the spinors must follow:

$$(\not{p} + m)u(\vec{p}) = 0$$

$$(-\not{p} + m)v(\vec{p}) = 0$$

- These equations have two solutions, which we label by + and -.

Two Solutions

- There are two solutions for u and two solutions for v.
  - We will distinguish these solutions by the eigenvalue of the spin matrix.
    - For u, we want the positive eigenvalue to correspond to u_+
    - For v, we want the positive eigenvalue to correspond to u_-
  - Why this unusual choice? As we'll show in problem 39.2, this results in the following commutation relation:

$$[J_z, b^\dagger_\pm(0)] = \pm\frac{1}{2} b^\dagger_\pm(0)$$

$$[\mathrm{J}_z, d^\dagger_\pm(0)] = \pm\tfrac{1}{2} d^\dagger_\pm(0)$$

    - This is nice because it implies that operators with daggers create spin-up particles.
    - Why? Loosely speaking, measuring J_z after acting with a dagger operator is the same as measuring it before acting with that operator, except for a difference of .5*operator. This "difference" term means that the operator must create an eigenstate of the J_z operator. The sign tells us that that the spin must be spin-up.

Calculating the Spinors

- Working in the rest frame, it is easy to solve the Dirac equation, and calculate an explicit form of the spinors (see equation 38.6).
  - We can then boost our result to whatever frame we like
  - We can also define barred spinors in analogy with the barred field operator from the previous chapter.

Spin Sums

- When we multiply the spinor by the barred spinor, we get a four by four matrix
- Next, let's sum over all spins.
  - This is really a sum over eigenstates of S_z
  - Thus, there should be no memory of the spin in the answer: it should depend only on the momentum and gamma matrices.
  - We also have an explicit form of the spinors for p = 0. We use this and generalize based on our conclusions above. The result is:

$$\sum_{s=\pm} u_s(p)\overline{u}_s(p) = -\not{p} + m$$

$$\sum_{s=\pm} v_s(p)\overline{v}_s(p) = -\not{p} - m$$

Helicity

- Helicity: the component of the spin in the direction of the three-momentum.
  - A fermion with helicity +1/2 is called right-handed
  - A fermion with helicity -1/2 is called left-handed

- The spinor corresponding to a right-handed fermion is:
  - $u_+(p)$ for a b-type particle
  - $v_-(p)$ for a d-type particle

  This is because the projection operator will map these onto the lower two components of the Dirac field, corresponding to the right-handed Weyl field.