QFT

Chapter 32: Spontaneous Breaking of Continuous Symmetries

Continuously Broken Symmetry

- This time we'll consider the Lagrangian given by:

$$\mathcal{L} = -\partial^\mu \phi^\dagger \partial_\mu \phi - m^2 \phi^\dagger \phi - \frac{1}{4}\lambda(\phi^\dagger \phi)^2$$

- Which is invariant under the U(1) transformation:

$$\phi(x) \longrightarrow e^{-i\alpha}\phi(x)$$

- If (and only if) $m^2 < 0$, then there is a whole family of minima in the potential, given by:

$$\phi(x) = \frac{1}{\sqrt{2}} v e^{-i\theta}$$

where

$$v = \sqrt{\frac{4|m^2|}{\lambda}}$$

The Goldstone Boson

- Now the argument goes as before:
  - choose a given ground state, say θ = 0.
  - switch our fields from φ and $φ^+$ to a and b
    - where φ = v + a + ib, as required to meet the renormalization condition < 0 | φ | 0 >
  - rewrite the Lagrangian in terms of the new fields
  - the new Lagrangian shows that the symmetry is broken

$$\mathcal{L} = -\frac{1}{2}\partial^\mu a \partial_\mu a - \frac{1}{2}\partial^\mu b \partial_\mu b - |m^2| a^2 - \frac{1}{2}\sqrt{\lambda}|m| a(a^2 + b^2) - \frac{1}{16}\lambda(a^2 + b^2)^2$$

- As before, we have two particles (a and b). One of these is massless: this is called the Goldstone Boson.
  - This is an example of Goldstone's Theorem

Goldstone's Theorem

- Goldstone's Theorem: If a symmetry (or a symmetry generator, in the non-Abelian case) is spontaneously broken (the ground state is not invariant under the action of the charges corresponding to the conserved currents, but rather shifts to a different vacuum), a massless particle must exist.

- This is independent of the parameterization chosen. We see the same behavior with this parameterization, in which the Goldstone boson does not appear in the potential, but rather only parameterizes the flat direction.

$$\mathcal{L} = -\frac{1}{2}\partial^\mu \rho \partial_\mu \rho - \frac{1}{2}\left(1 + \frac{\rho}{v}\right)^2 \partial^\mu \chi \partial_\mu \chi - |m^2|\rho^2 - \frac{1}{2}\sqrt{\lambda}|m|\rho^3 - \frac{1}{16}\lambda\rho^4$$

More observations

- The exact propagator should have a pole at $k^2 = 0$.
  - This means (by eqn. 14.6) that $\Pi(0) = 0$
  - We evaluate $\Pi(0)$ by summing all 1PI diagrams with two external lines, each with four momentum 0. But the vertex factors are proportional to the momentum of the external lines (due to the derivative) – and so they vanish.
  - So the goldstone boson really is massless.

- All this can also be deduced from considering the quantum action
  - The quantum action has the same symmetries as the classical action.
  - Spontaneous symmetry breaking occurs if Γ is at a constant, nonzero value of φ.
  - The phase of this constant is arbitrary. So, there must be a flat direction in field space, corresponding to the phase
  - The physical consequence of this flat direction in field space is the massless particle.

The Nonabelian Case

- What if the symmetries don't commute? For example, you could have a Lagrangian invariant under this transformation:

$$\delta\phi_i = -i\theta^a (T^a)_{ij}\phi_j$$

- As usual, if $m^2 < 0$, the minimum of the potential is achieved at $\langle 0|\varphi|0\rangle$, the vacuum expectation value (VEV).
  - The VEV is changed under an SO(N) transformation.
  - This change is zero if the corresponding T has only zeros in the last column
  - These Ts are called broken: Tv is nonzero.
  - An infinitesimal SO(N) transformation changes the VEV of the field, but not the energy – this means a flat dimension in phase space, and a massless Goldstone boson.
  - Unbroken generators don't change the VEV, so they should still be a manifest symmetry. The number of unbroken generators (.5[N-1][N-2]) corresponds to SO(N-1), which should therefore be an obvious symmetry of the Lagrangian, after it is rewritten in terms of shifted fields.