QFT

Chapter 31: Broken Symmetry and Loop Corrections

Overview

- In the previous section, we used the quantum action to show that the Zs in the “usual” Lagrangian are sufficient to cancel the divergences in the spontaneously-broken-symmetry Lagrangian, despite the necessity of additional terms.

- In this section, we use φ^4 theory as an example at one-loop level, to verify that this does indeed happen. We adjust the Lagrangian as before (breaking the symmetry):

$$\mathcal{L} = -\frac{1}{2}Z_\phi \partial^\mu \rho \partial_\mu \rho - \frac{1}{2}(\frac{3}{2}Z_\lambda - \frac{1}{2}Z_m)m_\rho^2 \rho^2 + \frac{1}{2}(Z_m - Z_\lambda)(3/\lambda\tilde{\mu}^{\epsilon 1/2} m_\rho^3 \rho$$
$$-\frac{1}{6}Z_\lambda (3\lambda\tilde{\mu}^\varepsilon)^{1/2} m_\rho \rho^3 - \frac{1}{24}Z_\lambda \lambda \tilde{\mu}^\varepsilon \rho^4$$

$\langle 0|\rho|0\rangle$

- In chapter 9, we showed that $\langle 0|\rho|0\rangle$ is given by the sum of all connected diagrams with one external line.

- We can draw two such diagrams, and assign the values using the Feynman Rules.
  - The Zs from here ($m^2 < 0$) are the same as from the unbroken Lagrangian ($m^2 > 0$).
  - The result is that the divergences have cancelled.

- Our renormalization condition is that this should equal zero, which we can impose by choosing the numerical constants appropriately.
  - This also cancels all one-loop tadpole diagrams.

Calculations & Conclusions

- We construct the self energy, V_3 function, and V_4 function in the same manner.
  - The procedure for this is the same as in problem 28.3
  - The result is that the divergent terms cancel.

- We conclude, as expected, that the Zs for a normalizable theory with an unbroken symmetry are also sufficient to cancel the divergences in the corresponding broken symmetry, regardless of the nature of the symmetry group.