QFT

Chapter 30: Spontaneous Symmetry Breaking

φ⁴ Theory, again

- Consider φ^4 theory:

$$\mathcal{L} = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{1}{24}\lambda \phi^4$$

- Some obvious observations:
  - We can swap φ --> -φ
  - In operator form, we can define $Z^{-1}\ \varphi\ Z = -\varphi$
  - Then $Z^2 = 1$, which implies $Z = Z^{-1}$
  - By definition, Z is unitary, so $Z^{-1} = Z^H$
  - This implies $Z = Z^H$, ie Z is Hermitian

- We can write this as:

$$\mathcal{L} = -\frac{1}{2}\partial^\mu \phi \partial_\mu \phi - \frac{1}{24}\lambda(\phi^2 - v^2)^2 - \frac{1}{24}\lambda v^4$$

where $v = \sqrt{6|m^2|/\lambda}$. The last term is constant (at least, it doesn't have any φs), so we drop it.

Ground State

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{24}\lambda\phi^4$$

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{24}\lambda(\phi^2 - v^2)^2$$

- What is the ground state of this Lagrangian?
  - If $m^2 > 0$, then we can easily see from the original Lagrangian that there is only one ground state at $\varphi = 0$. Then $\langle 0|\varphi|0\rangle = 0$, just as required by the renormalization scheme.
  - If $m^2 < 0$, then we see from the new Lagrangian that the ground is clear: $\varphi = \pm v$.

- In QFT also (with $m^2 < 0$), we expect two ground states, 0+ and 0-. These are exchanged by the operator 0, and are orthogonal.
  - Why orthogonal? Consider space as a collection of harmonic oscillators. You can tunnel from 0+ to 0-, but all infinitely many would have to do so, which lowers the amplitude to 0.

Broken Symmetry

- Let's imagine that 0+ is the ground state (when m < 0). Let's also define ρ = φ – v. This is required to fix our renormalization condition <0|ρ|0>.

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{6}\lambda v^2\rho^2 - \frac{1}{6}\lambda v\rho^3 - \frac{1}{24}\lambda\rho^4$$

- The second term coefficient could be set to ½ m_ρ^2, the mass of the ρ particle.

- But there is a problem: where the hell is our Z_2 symmetry?
  - Spontaneously broken!

Symmetry Breaking in the Quantum Action

- Recall in problem 21.2, we showed that the quantum action inherits any linear symmetry of the classical action, provided that it is also a symmetry of the integration measure (which it usually is).
- The quantum action “therefore” shows the spontaneous symmetry breaking as well.
- All the ingredients of $\Gamma(\varphi)$ are made finite with the three existing Zs, so no additional Zs are needed.
- We’ll see how this works in the next section.