QFT:

Chapter 29: Effective Field Theory

Comment

- This is perhaps the worst chapter in Srednicki. He seems to alternate between being fairly detailed (pg 179-180) and very general (where did equation 29.11 come from?)

- I'm going to focus here on the conceptual issues. The mathematics is so poorly explained that we can't really understand much, and is ultimately unimportant going forward.

Review

- Recall our discussion of non-renormalizable theories.
  - If the Zs can absorb all the divergences, our theory is renormalizable.
  - If not, but we can absorb the divergences by adding a finite number of terms to the Lagrangian, then our theory is still renormalizable.
  - If we have to add an infinite number of terms to the Lagrangian to absorb this, then our theory is non-renormalizable.

- But, nonrenormalizable field theories can still be useful. We take up their analysis here.

Euclidean Space

- First, we do a Wick Rotation directly on the action.
  - The new action is called the Effective Action
  - The advantage here is that every term in S_E is nonnegative, and so the path integral is strongly damped, not rapidly oscillating.
  - Note that in Euclidean space, the kinetic term in the Lagrangian is doubly covariant:

$$\partial_\mu \phi \partial_\mu \phi = (\partial \phi / \partial \tau)^2 + (\nabla \phi)^2$$

- Next we switch to momentum space.

Ultraviolet Cutoff

- The problem is we have these divergent integrals, so we install an upper bound Λ on the integral.
  - This ultraviolet cutoff should be much larger than any energy scale of practical interest.
  - Recall that we switched to momentum space, so these momentum integrals are now cut off at this scale also.

Infinite Number of Diagrams

- Our effective action as shown in equation 29.11 now has an infinite number of terms in it (why?).
  - These terms correspond to all possible diagrams with high dimensionality and the same symmetries as the original Lagrangian.

- If we evaluate one of these terms, we find:
  - m^2 is the only coefficient with positive mass dimension, and is dominated by contributions from the high end of the integral.
  - Terms with zero mass dimension is dominated equally over the integration range (such as λ in φ^4 theory).
  - Terms with negative mass dimension are most dominated by the lower end of the integral.

Naturalness

- We need $m^2(\Lambda)$ to to be around m^2_{ph}.
  - There are many terms of $O(\Lambda^2)$. We need these to sum to meet this condition
  - So now we have an infinite number of diagrams, that need to sum to give a very precise answer.
  - Further, Λ is huge and m is relatively tiny, so this cancellation has to be absolutely perfect (down to several decimal places).
  - This problem is called the naturalness problems.

- For the moment, we will accept that the fine-tuning has taken place.

- In higher-spin fields, the action has more symmetry, and this presents divergences that are worse than logarithmic.
  - This is called Technical Naturalness
  - Technical naturalness can also be used for supersymmetry – we'll talk more about that in chapter 95)

Beta Functions

- To understand the lower-energy behavior of non-renormalizable (or renormalizable) theories, we can integrate between Λ (some lower energy scale) and Λ_0 (the UV cutoff), then take the derivative to rederive the beta function.
  - We'll do this in detail in problem 29.2.

Summary: Wilson's Method

- Putting all this together (sort of), we see the general approach (maybe):
  - Put in an explicit UV cutoff
  - Insert a lower cutoff, and do the integral between the two cutoffs
  - This will change the coefficients in the effective action
  - The coefficients with negative mass dimension will start to take on the same values that we would have calculated in perturbation theory (where only renormalizable theories can be trusted)
  - Continuing to rescale until the kinetic terms become canonical, the dimensionless coupling constants change according to their beta functions
  - And so, at low energies, the exact answer is the same as the perturbative answer, up to small corrections by powers Λ/Λ_0

Remove UV cutoff, and Triviality

- So that's low energies. What about high energies? Can we remove this UV cutoff?

- We can try to integrate the renormalization-group equation using the exact beta function, then take the limit as the cutoff goes to infinity.
  - But, Srednicki tries to do exactly this for φ^4 theory, and finds that he instead derives a maximum value for the initial cutoff.
  - Thus, we cannot take the limit as the cutoff goes to infinity; we must instead use an effective field theory.

- Can we insist? Let's take the limit in any case!
  - This will force the coupling to vanish – at all energy scales! This is called triviality.

- But this is only for beta functions that grow quickly (faster than coupling).
  - If beta functions grows slower than coupling, will end up with a ultraviolet fixed point, a sort of constant, minimum effective coupling.
  - If beta function is negative, the theory is asymptotically free – no coupling at higher energy scales. This is of course desirable, but turns out to be possible only for non-Abelian gauge theories.