QFT:

Chapter 28: The Renormalization Group

Overview

- We repeat the analysis of the previous section in a more formal way

- Equation that tell us how the non-directly-measurable quantities (like correlation functions) vary with μ are collectively called the equations of the renormalization group

Renormalized Fields and Bare Fields

- The Lagrangian for the φ^3 theory can be written in two ways: renormalized and bare:

$$\mathcal{L} = -\frac{1}{2}Z_\phi \partial^\mu \phi \partial_\mu \phi - \frac{1}{2}Z_m m^2 \phi^2 + \frac{1}{6}Z_g g \tilde{\mu}^{\varepsilon/2}\phi^3 + Y\phi$$

$$\mathcal{L} = -\frac{1}{2}\partial^\mu \phi_0 \partial_\mu \phi_0 - \frac{1}{2}m_0^2 \phi_0^2 + \frac{1}{6}g_0 \phi_0^3 + Y_0 \phi_0$$

- By comparing the equations, it is possible to relate the renormalized parameters with the bare parameters.

Z Expansion

- Recall that in the MS bar renormalization scheme, the Zs are chosen to cancel the infinites in the loop integrals (and nothing more). Thus, we can write them as:

$$Z_i = 1 + \sum_{n=1}^{\infty} \frac{a_n(\alpha)}{\varepsilon^n}$$

- We determine these by calculating the self-energy and the vertex function in the MS bar scheme, choosing the Zs to cancel the infinities.

G

- In principal, the theory is completely specified by the bare parameters.
  - Since the exact scattering amplitudes are independent of μ, it follows that the bare parameters must be as well.

- Now recall that:

$$\alpha_0 = \frac{g_0^2}{(4\pi)^3}$$

  - We can rewrite the bare field g_0 with the renormalized fields.
  - It is convenient to define G as the natural log of the renormalized fields in this equation.
  - We'll also define G as a Taylor Series in inverse powers of ε.

Conclusions

- Now that we've defined G, it is a simple matter to play with the expansions in order to determine the beta function and anomalous dimensions.
  - Of course, the numbers match those of the previous section.

- By relating the scalar propagator with the bare-field scalar propagator, we can derive the Callen-Symanzik equation for the propagator.
  - The result is that the propagator is proportional to k^{-2+C}, where C is related to the anomalous dimension of the field
  - This has applications in the theory of critical phenomena (melting, for example)