QFT

Chapter 27: Other Renormalization Schemes

Overview

- Recall that our expression for the amplitude still diverges at low m. Here we fix that.
- Recall that in our expression for Π, there are two free parameters, A and B.
  - These were fixed by requiring $\Pi(-m^2) = 0$ and $\Pi'(-m^2) = 0$.
- But in the massless limit:
  - $\Pi(-m^2) = 0$ for all A, B.
  - $\Pi'(-m^2)$ is not well defined.
- Physically, the problem is that the one-particle states are not separated from the multiparticle continuum by a finite gap in energy (as in the previous section).

Renormalization Schemes

- We have until now required on shell (OS) renormalization
  - The exact propagator has a pole at $k^2 = -m^2$, with residue one
- A new option is modified minimal-subtraction (MS bar) renormalization.
  - Minimal Subtraction is designed such that A and B cancel the infinity term in the self-energy (only).
  - Modified Minimal Subtraction is the same thing, but with mu tilde defined as before, rather than having mu = mu tilde.
- Advantages
  - The MS bar scheme is well defined in the massless limit
  - The OS scheme gives a self-energy that does not depend on mu

Consequences of MS bar Renormalization Scheme

- The propagator will no longer have its pole at $k^2 = -m^2$; it will be somewhere else.
  - By definition, the physical mass of the particle is at the new location; $k^2 = -m_{ph}^2$.
  - So, the Lagrangian Parameter m no longer corresponds to the physical mass m_{ph}.

- The residue of the pole is no longer one.
  - The field $R^{-1/2}\varphi$ has unit amplitude to create a one particle state.
  - So, LSZ formula must be corrected by adding $R^{-1/2}$ for each external particle.
  - Further, the Klein-Gordon wave operator hits each external propagator and cancels the momentum-space pole, leaving a residue R.
  - The net effect is a factor of $R^{1/2}$ for each external particle.

- The m in the LSZ formula corresponds to the physical mass, not the Lagrangian parameter.

m and m_{ph}

- Recall the exact form of the propagator:

$$\Delta_{\overline{MS}}(k^2)^{-1} = k^2 + m^2 - \Pi_{\overline{MS}}(k^2)$$

- And that by definition:

$$\Delta_{\overline{MS}}(-m_{ph})^{-1} = 0$$

- This gives:

$$m_{ph}^2 = m^2 - \Pi_{\overline{MS}}(-m_{ph}^2)$$

- Since Π is O(α), we can write this as:

$$m_{ph}^2 = m^2 - \Pi_{\overline{MS}}(-m^2) + O(\alpha^2)$$

- which is (for φ^3 theory):

$$m_{ph}^2 = m^2\left[1 + \frac{5}{12}\alpha\left(\ln(\mu^2/m^2) + \frac{34 - 3\pi\sqrt{3}}{15}\right) + O(\alpha^2)\right]$$

Anomalous Dimension

$$m_{ph}^2 = m^2 \left[1 + \frac{5}{12}\alpha \left(\ln(\mu^2/m^2) + \frac{34 - 3\pi\sqrt{3}}{15}\right) + O(\alpha^2)\right]$$

- This should be independent of μ!
  - To impose this, we need α and m to depend on μ in such a manner that m_{ph} is left invariant.
  - This manner is called the anomalous dimension of the mass parameter, defined as follows:

$$\gamma_m(\alpha) = \frac{1}{m}\frac{dm}{d\ln\mu}$$

Residue of (new) pole, and V_3

- The residue is defined by:

$$R^{-1} = \frac{d}{dk^2}\left[\Delta_{\overline{MS}}(k^2)^{-1}\right]\Bigg|_{k^2=-m_{ph}^2}$$

- Doing the calculation, we find:

$$R^{-1} = 1 + \frac{\alpha}{12}\left[\ln(\mu^2/m^2) + \frac{17-3\pi\sqrt{3}}{3}\right] + O(\alpha^2)$$

- We can also use MS bar to find V_3:

$$V_{3,MS}(k_1,k_2,k_3) = g\left[1 - \frac{\alpha}{2}\int dF_3 \ln(D/\mu^2) + O(\alpha^2)\right]$$

φφ --> φφ Scattering Amplitude

- Recalculating V_3, we remember to:
  - Include LSZ correction factor from previous section
  - Multiply by correction factor that accounts for angular resolution of the detector
  - Use vertex function and propagators from new renormalization scheme.

- We find that:

$$|\mathcal{T}|^2_{\text{obs}} = |\mathcal{T}_0|^2 \left[1 - \alpha\left(\frac{3}{2}\ln(s/\mu^2) + \frac{1}{3}\ln(1/\delta^2) + O(m^0)\right) + O(\alpha^2)\right]$$

- At last, this is well-defined in the m --> 0 limit.

Beta Functions

- As before, we need the μ dependence of α to be cancelled by the explicit μ dependence of the amplitude, leaving the amplitude invariant.

- This dependence on dimensionality is defined by the beta function:

$$\beta(\alpha) = \frac{d\alpha}{d\ln\mu}$$

- Calculating our beta function for φ^3 theory, we find that

$$\frac{d\alpha}{d\ln\mu} = -\frac{3}{2}\alpha^2$$

Asymptotic Freedom

- Solving this, we find that:

$$\alpha(\mu_2) = \frac{\alpha(\mu_1)}{1 + \frac{3}{2}\alpha(\mu_1)\ln(\mu_2/\mu_1)}$$

- Notice that as μ decreases, α increases. This is called asymptotic freedom.
  - Remember that α is related to g; and μ has dimensions of energy. So, the system becomes more and more strongly coupled at lower energies.
  - Put differently, the tree-level result is better and better at higher energies.
- For massive particles, we should not choose μ << m.
  - s has a minimum at $4m^2$, so μ below that will lead to a large log.
  - If α << 1, then it's fair to use perturbation theory at these low energies.
- For massless particles, α continues to increase at lower energies
  - Hence, the terms with many factors of g are the most important ones. This is the opposite of perturbation theory, so perturbation theory breaks down.
  - So, low energy physics may be quite different than what perturbation theory predicts.

Infrared Freedom

- φ^3 theory is therefore asymptotically free.
  - Hence, our well-defined results don't necessarily work at low energy.
  - That's good, because in this case, we know the correct low-energy result: the particle will tunnel through the potential barrier and then lose energy indefinitely.
    - This is why the "real world" couldn't possibly follow φ^3 theory, as discussed before.

- What if we have asymptotic freedom in a system with a ground state?
  - If the sign of the beta function is positive, then the coupling increases as μ increases. So, perturbation theory breaks down if the energy is too high.
  - If the sign of the beta function is not always positive, then more complicated behaviors can result. We'll discuss some of these in the next section.