QFT

Chapter 25: Unstable Particles and Resonances

Overview

- In problem 11.1, we calculated the decay rate of one real particle A into two real particles B (assuming that $m_A > 2m_B$) in φ^3 theory.

- In chapter 18, we learned that this should be done in six dimensions rather than 4, in order to have a renormalizable theory. We know how to repeat the calculation in 6 dimensions.

- The problem is that the LSZ formula requires incoming and outgoing particles to correspond to exact eigenstates of the Hamiltonian. This can't be a decaying particle!

- Thus, the LSZ formula is unacceptable for nonstable particles. In this section, we find the decay rate without the LSZ formula.

One-Loop Correction

- Let's turn aside from this problem for the moment to do something seemingly unrelated: the one-loop correction. We'll focus on the A loop, assuming that the B loop is much weaker.

- We do this in the usual way, but find something remarkable. The self-energy is usually real (why?) but this time the integration limits cause Π to have an imaginary part given by:

$$\mathrm{Im}\ \Pi(-m_A^2) = m_A \Gamma$$

- This is a general result, and it's worth understanding why.

Argument 1: “Physical Intuition”

- You can imagine Γ as the average inverse lifetime of the A particle. This follows from having the metastable state as a resonance in the partial wave amplitude:

$$f_\ell(E) \propto \frac{1}{E - E_0 + i\Gamma/2}$$

- The Lehmann-Källén formula tells us that the exact propagator of the A has a pole at $k^2 = -m_A{}^2$, ie when
  $s = m_A{}^2$.

- This means that where $s \approx m_A{}^2$, the BB scattering amplitude is dominated by s-channel A exchange. Then:

$$\mathcal{T} \approx \frac{g^2}{-s + m_A^2 - \Pi(-s)}$$

Argument 1: “Physical Intuition”

- If we’re off shell by a small amount ε, then this simplifies to:

$$\mathcal{T} \approx \frac{g^2/2m_A}{\varepsilon + \Pi(-m_A^2)/2m_A}$$

- Comparing this to our first result:

$$f_\ell(E) \propto \frac{1}{E - E_0 + i\Gamma/2}$$

- We find that:

$$\mathrm{Im}\ \Pi(-m_A^2) = m_A \Gamma$$

Argument 2: Proof Sketch

- Srednicki works through the proof, which I will outline here:
  - Write full expression for self-energy
  - Take imaginary part using identity of eqn. 15.7
  - Use advanced/retarded propagators rather than Feynman propagators to simplify
  - Substitute in dLIPS, and take value for on-shell A particle.

- By the way, this argument can be generalized to give the Cutkosky rules for computing the imaginary part of any Feynman Diagram (not necessary at this level)