QFT

Chapter 23: Discrete Symmetries: P, T, C, Z

Recall

- Mathematically, a Lorentz Transformation is anything that preserves the interval.
- We decided in section 2 to restrict Lorentz Transformations to be proper and orthochronous. We decided that our QFT had to be invariant under these Lorentz Transformations only.
- Recall that this implies that QFT is not, in general, invariant under discrete transformations
  - In this section we turn to the exceptions: discrete transformations under which QFT should be invariant.

Parity and Time-Reversal

- Parity: flip all spatial axes

  P = diag(1, -1, -1, -1)

- Time Reversal: flip axis of time

  T = diag(-1, 1, 1, 1)

- Are there unitary operators that can impose these symmetries, just like for proper orthochronous transformations? Let's assume that there are. Then we'd expect:

$$P^{-2}\phi(x)P^2 = \phi(x)$$
$$T^{-2}\phi(x)T^2 = \phi(x)$$

because two such transformations should undo one another.

Even and Odd

- To be consistent with this requirement, the definition of our unitary operators could be either of the following.

$$P^{-1}\phi(x)P = \phi(\mathcal{P}x) \qquad P^{-1}\phi(x)P = -\phi(\mathcal{P}x)$$

$$T^{-1}\phi(x)T = \phi(\mathcal{T}x) \qquad T^{-1}\phi(x)T = -\phi(\mathcal{T}x)$$

- Why didn't this happen before?
  - For a continuous transformation, the field must be unchanged in the limit of no transformation. This is a discrete transformation.
- So which definition is right?
  - In principle, we get to choose.
  - But it's best to choose the one that makes the Lagrangian even, since that will lead to an invariant action (ie T and P are conserved).

Antiunitarity of T

- T is technically a Lorentz Transform, so we expect that:

$$T^{-1}P^{\mu}T = \mathcal{T}^{\mu}_{\ \nu}P^{\nu}$$

- For μ = 0, we have T^{-1} H T = -H
- Uh oh! We are invariant under time reversal only if H = 0. Let's go back to the infinitesimal version:

$$T^{-1}(I - ia_{\mu}P^{\mu})T = I - i\mathcal{T}^{\mu}_{\ \nu}a_{\mu}P^{\nu}$$

- If we equate the coefficients of a_{mu} on both sides, we get back where we started (that's where our expectation comes from).
    - Instead, let's impose anti-unitarity, $T^{-1}iT = -i$. This fixes our problems.

Z Symmetry

- Alternatively, let's keep the spacetime arguments the same, but change the sign of the field. We'll call this Z-symmetry. Hence,

$$Z^{-1}\phi_a Z = \eta_a \phi_a(x)$$

where $\eta = \pm 1$ (can be different for each field).

- Obviously $Z^2 = I$ and $Z = Z^{-1}$.

- We'll call this a Z_2 operator, referring to the group (addition modulo 2, which corresponds to multiplying by 1 or -1).

Charge Conjugation & Antiparticles

- As a special case of Z, we can set all imaginary fields negative, while keeping real fields positive. This is called complex conjugation, or C symmetry.

- Imposing a C transformation swaps the a-type particles for the b-type particles in a complex theory. The signs are reversed, but the masses are not. We therefore interpret these as antiparticles.