QFT

Chapter 22: Continuous Symmetries and Conserved Currents

Overview

- We now turn to our next topic: symmetries…
  - Continuous Symmetries (ch. 22)
  - Discete Symmetries (ch. 23)
  - Non-Abelian Symmetries (ch. 24)

- At the end of the part 1, we'll address symmetry breaking, which will help us construct the standard model in part 3

Transformation of the Lagrangian

- Let's transform:

$$\phi_a(x) \to \phi_a(x) + \delta\phi_a(x)$$

- The Langrangian is therefore transformed as:

$$\delta\mathcal{L}(x) = \frac{\partial\mathcal{L}}{\partial\phi_a(x)}\delta\phi_a(x) + \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_a(x))}\partial_\mu\delta\phi_a(x)$$

- Next, we use the field equation to find that:

$$0 = \frac{\delta S}{\delta\phi_a(x)} = \frac{\partial\mathcal{L}(x)}{\partial\phi_a(x)} - \partial_\mu\frac{\partial\mathcal{L}(x)}{\partial(\partial_\mu\phi_a(x))}$$

Note that the first equality holds only if the classical field equations are satisfied.

The Noether Current

- Using these results, we can rewrite the infinitesimal Lagrangian as:

$$\delta\mathcal{L}(x) = \partial_\mu \left(\frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \phi_a(x))} \delta\phi_a(x) \right) + \frac{\delta S}{\delta \phi_a(x)} \delta\phi_a(x)$$

- The Noether current is the portion in large parenthesis.

- The Noether current is conserved if:
  - The field equations are satisfied
  - The Langrangian is invariant under the transformation.

- The upshot of this is that we get a new conservation law:

$$\partial_\mu j^\mu = 0$$

The Noether Charge

- The integral of the Noether current is the Noether charge
  - This must be constant in time, assuming reasonable boundary conditions

- By decomposing the field operators and creation/ annihilation operators, we find that:

$$Q = \int \widetilde{dk}\, [a^*(k)a(k) - b(k)b^*(k)]$$

which is the number of A particles minus the number of B particles. So the A and B particles have opposite Noether charge but the same mass. Sounds like antiparticles! More on this in next chapter…

Utility of the Noether Current

- We can use these ideas to derive some identities:
  - Schwinger-Dyson
  - Ward

  These identities are rather mathematical, so I won't state or prove them here.

- If the differential change in the Lagrangian is nonzero, but rather a total divergence of a function K, there is still a conserved current:

$$j^\mu(x) = \frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu \phi_a(x))}\delta\phi_a(x) - K^\mu(x)$$

- In the case of a spacetime translation, for example, we find that the Noether current is related to the stress-energy tensor, which is highly useful.

Lorentz Symmetry

- An infinitesimal Lorentz Transformation is another symmetry that has a conserved current associated with it. The conserved current is:

$$\mathcal{M}^{\mu\nu\rho}(x) = x^{\nu}T^{\mu\rho}(x) - x^{\rho}T^{\mu\nu}(x)$$

- The conserved charge is:

$$M^{\nu\rho} = \int d^3x \mathcal{M}^{0\nu\rho}(x)$$

- These are the generators of the Lorentz Group that were introduced before.
  - We can check this using the canonical commutation relations (see problem 22.3).