# QFT

Chapter 20: Two-Particle Elastic Scattering at One Loop

# Overview

- This is an example of the last section, in which we discussed how to combine our corrections to calculate the scattering amplitude to arbitrary order in g.
- Our example will be elastic (particle-number conserving) two-particle scattering in $\varphi^3$ theory in 6 dimensions

# The Diagrams

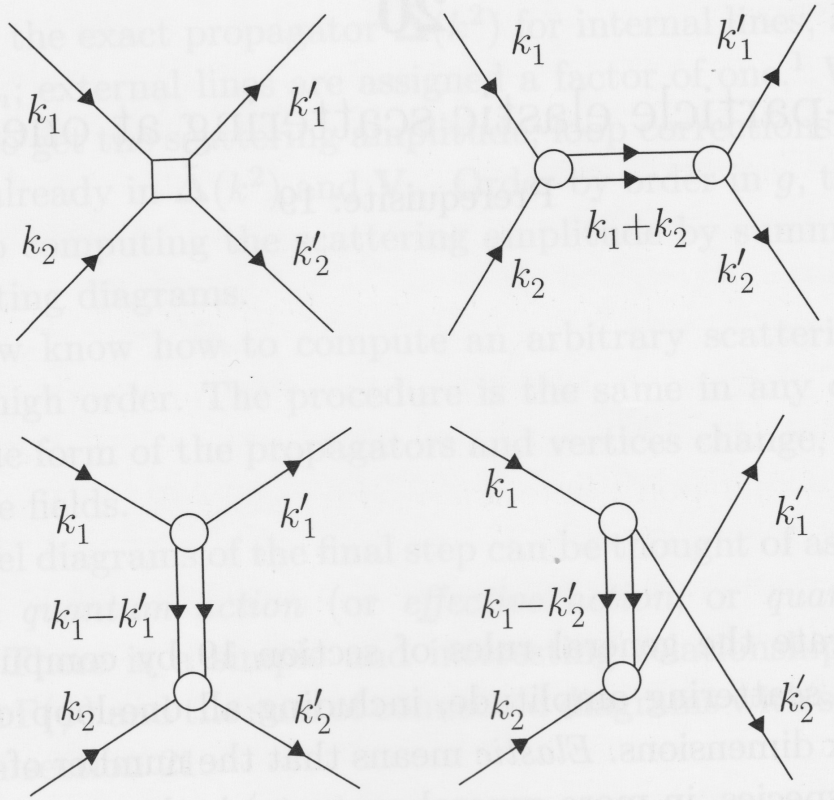


Figure 20.1. The Feynman diagrams contributing to the two-particle elastic scattering amplitude; a double line stands for the exact propagator $\frac{1}{i}\tilde{\mathbf{\Delta}}(k)$, a circle for the exact three-point vertex $\mathbf{V}_3(k_1, k_2, k_3)$, and a square for the exact four-point vertex $\mathbf{V}_4(k_1, k_2, k_3, k_4)$. An external line stands for the unit residue of the pole at $k^2 = -m^2$.

# The Scattering Amplitude

- Previously, we determined that the scattering amplitude at tree-level is given by:

$$i\mathcal{T} = \frac{1}{i}(ig)^2\left[\tilde{\Delta}(-s) + \tilde{\Delta}(-t) + \tilde{\Delta}(-u)\right]$$

- Thanks to first-order corrections, we have a new diagram (the four-point vertex) as well as a more complicated vertex factor ($iV_3$ rather than ig). Then:

$$i\mathcal{T} = \frac{1}{i}\left[[i\mathbf{V}_3(s)]^2\tilde{\boldsymbol{\Delta}}(-s) + [i\mathbf{V}_3(t)]^2\tilde{\boldsymbol{\Delta}}(-t) + [i\mathbf{V}_3(u)]^2\tilde{\boldsymbol{\Delta}}(-u) + i\mathbf{V}_4(s,t,u)\right]$$

where the boldface denotes exact functions.

# Results

- $V_3(k_1, k_2, k_3)$ is given. We take
  $$V_3(s) = V_3(-m^2, -m^2, -s)$$
  - You can think of this as two incoming particles on shell, and one outgoing particle that conserves momentum.
  - Similar for $V_4$

- From here it's just a matter of doing the calculus. The result is that **the original tree-level amplitude is corrected by powers of logarithms of kinematic variables.**