QFT

Chapter 19: Perturbation Theory to all Orders

Overview

- We've seen that loops can correct both the propagator and the vertex.
- But how do we put this all together to calculate the scattering amplitude?
  - In this chapter, we talk about the theory
  - In the next chapter, we conclude our study of loop-corrections by calculating the scattering amplitude to one-loop order.

All Corrections

- Summing the 1PI diagrams with two external lines gives us the self-energy.
- Summing the 1PI diagrams with three external lines gives us V_3.
- We must adjust coefficients of Zφ, Zm, and Zg to maintain the conditions $\Pi(-m^2) = 0$, $\Pi'(-m^2) = 0$, and $V_3 = g$.
- We also need the n-point vertex function, from V_4 up to V_E, where E is the number of external lines in the process, since all of these vertex corrections contribute.
  - One option is to sum all the contributing 1PI diagrams
  - Equivalently, we can sum over skeleton diagrams

Skeleton Diagrams

- Skeleton Expansion: draw all diagrams that contribute to V_n, but omit those that include propagator or three-point vertex corrections.
  - ie, omit diagrams that contain a subdiagram with two or three external lines that is more complicated than a tree level propagator or vertex.
- Then, take the propagators and vertices in these diagrams to be given by the exact propagator and vertex V_3, rather than by the tree-level propagator and vertex g.
- Sum these skeleton diagrams to get V_n. Order by order in g, this is equivalent to summing the usual set of 1PI diagrams.

Calculating the Scattering amplitudes

- Next, draw all tree-level diagrams, including those with n-point vertices for $3 \leq n \leq E$.
  - Again, use the exact propagator for internal lines, and exact V_n.
  - Sum these tree diagrams to get the scattering amplitudes. Loop corrections can be ignored, since they were accounted for in the exact propagators and V_ns.

- Great! We're done. But notice we can think about this differently. We can imagine that there *are* no corrections, and that the tree-level diagrams are the only diagrams for a different field theory, one that has action $\Gamma(\varphi)$ – as opposed to the normal field theory, which has action $iW(J)$.

- After an example in chapter 20, we'll turn to effective field theory, which explores the relationship between $\Gamma(\varphi)$ and $W(J)$.