QFT

Chapter 18: Higher-order corrections and renormalizability

Renormalizability

- Renormalizability has to do with divergences in the values of the diagrams.
  - If there are no divergences, then the theory is renormalizable
  - Otherwise, can the divergences be absorbed into the coefficients of terms in the Lagrangian? If so, then renormalizable.
  - Otherwise, can a finite number of new terms be added to the Lagrangian to cancel the divergences? If so, then renormalizable.
  - Otherwise, nonrenormalizable.

- Nonrenormalizable theories can often make useful predictions below an ultraviolet cutoff. More on this later.

Condition for Renormalizability

- Can we look at a Lagrangian and determine whether it is renormalizable or not?

- Every Feynman diagram gives:
  - d integrals for each loop
  - A term proportional to l^{-2}, where l is the loop momentum, for each internal propagator.

- So we define the superficial degree of divergence to be

$$D = dL - 2I$$

where L is the number of loops and I is the number of internal propagators.

- If $D < 0$, then there are at least as many terms in the denominator as in the numerator, so everything is fine. Otherwise, the diagram is (at first inspection) divergent.
  - If $D = 0$, then there is an l^n in the denominator and and n integrals. But n integrals means only l^{n-1} in the numerator, so the integral still converges.

A more useful condition for Renormalizability

- The tree-level diagram has value $-iZ_E g_E$. All diagrams must have the same value (since they all go into the scattering amplitude in the same way).

- **Hence, the mass dimensionality of the diagram is the same as the mass dimensionality of the coupling constant g_E.**
  - Recall that the Z_Es are numbers, by definition.

- We can break apart the mass dimensionality of the diagram in terms of the mass dimensionality of the components.
  - Every loop integral contributes d
  - Every internal propagator contributes -2.
  - Every vertex contributes $[g_n]$, where n varies depending on the type of vertex.

A more useful condition for Renormalizability

- Putting this together, we have:

$$D = [g_E] - \sum_n V_n [g_n]$$

  where E is the type of vertex corresponding to the tree diagram, and n sums over all the different types of vertices (if n = 4, the vertex joins 4 external lines). V_n is the number of each type of vertex.

- If any $[g_n] < 0$, then we expect uncontrollable divergences, since we can draw an infinite number of diverging diagrams, forcing $D > 0$.

A more useful condition for Renormalizability

- But we know that $[g_n]$ depends on the number of dimensions and the number of particles being joined. Using our result from ch. 12, we have a divergence if:

$$n > \frac{2d}{d-2}$$

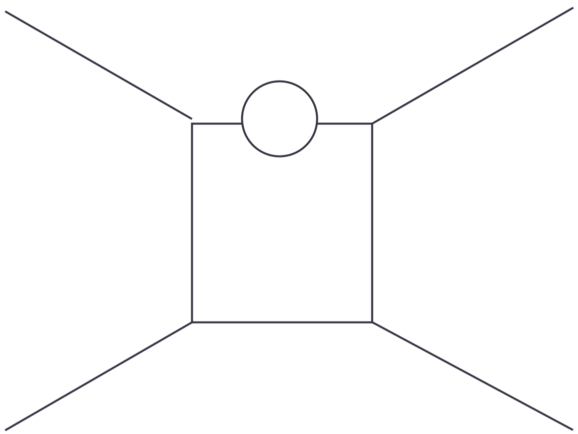
- Thus we are limited to:
  - Powers no higher than φ^4 in four dimensions
  - Powers no higher than φ^3 in six dimensions

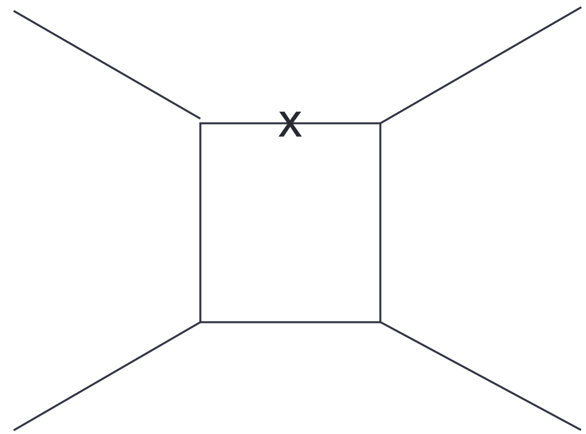
- Didn't we already know this?
  - We hinted at it before. This is our first time seeing that it would require an infinite number of corrections to cancel all the divergences.

- We call D the superficial degree of divergence, because there are exceptions to this rule.
  - If the loop momenta in the numerator cancel, we might get convergence even if $D \geq 0$.
  - If $D < 0$, we can still get divergence….

Divergences with D < 0

- Consider these diagrams:





  - Both the loop and the CT vertex are divergent, but these divergences cancel.
  - Even when the divergences don't cancel, Z factor can be adjusted to cancel these "divergent subdiagrams."
  - In both cases, the theory is renormalizable despite the divergence.

- Thus, if all couplings have nonnegative mass dimensions, the theory is renormalizable.
  - Proving this is very difficult, so we won't bother.
  - But, it turns out to be true for spin-0 and spin-1/2 fields.
  - It's true for spin 1 fields when there is an associated gauge symmetry
  - It turns out that theories with spin > 1 are never renormalizable for d ≥ 4.