Chapter 18: Higher-order corrections and renormalizability
Renormalizability

• Renormalizability has to do with divergences in the values of the diagrams.
  • If there are no divergences, then the theory is renormalizable
  • Otherwise, can the divergences be absorbed into the coefficients of terms in the Lagrangian? If so, then renormalizable.
  • Otherwise, can a finite number of new terms be added to the Lagrangian to cancel the divergences? If so, then renormalizable.
  • Otherwise, nonrenormalizable.

• Nonrenormalizable theories can often make useful predictions below an ultraviolet cutoff. More on this later.
Condition for Renormalizability

• Can we look at a Lagrangian and determine whether it is renormalizable or not?

• Every Feynman diagram gives:
  • d integrals for each loop
  • A term proportional to $l^{-2}$, where $l$ is the loop momentum, for each internal propagator.

• So we define the superficial degree of divergence to be
  \[ D = dL - 2I \]
  where $L$ is the number of loops and $I$ is the number of internal propagators.

• If $D < 0$, then there are at least as many terms in the denominator as in the numerator, so everything is fine. Otherwise, the diagram is (at first inspection) divergent.
  • If $D = 0$, then there is an $l^n$ in the denominator and and $n$ integrals. But $n$ integrals means only $l^{n-1}$ in the numerator, so the integral still converges.
A more useful condition for Renormalizability

• The tree-level diagram has value \(-iZ_E g_E\). All diagrams must have the same value (since they all go into the scattering amplitude in the same way).

• Hence, the mass dimensionality of the diagram is the same as the mass dimensionality of the coupling constant \(g_E\).
  • Recall that the \(Z_E\)s are numbers, by definition.

• We can break apart the mass dimensionality of the diagram in terms of the mass dimensionality of the components.
  • Every loop integral contributes \(d\)
  • Every internal propagator contributes -2.
  • Every vertex contributes \([g_n]\), where \(n\) varies depending on the type of vertex.
A more useful condition for Renormalizability

- Putting this together, we have:

\[ D = [g_E] - \sum_n V_n[g_n] \]

where \( E \) is the type of vertex corresponding to the tree diagram, and \( n \) sums over all the different types of vertices (if \( n = 4 \), the vertex joins 4 external lines). \( V_n \) is the number of each type of vertex.

- If any \( [g_n] < 0 \), then we expect uncontrollable divergences, since we can draw an infinite number of diverging diagrams, forcing \( D > 0 \).
A more useful condition for Renormalizability

• But we know that \([g_n]\) depends on the number of dimensions and the number of particles being joined. Using our result from ch. 12, we have a divergence if:
  \[
  n > \frac{2d}{d - 2}
  \]

• Thus we are limited to:
  • Powers no higher than \(\varphi^4\) in four dimensions
  • Powers no higher than \(\varphi^3\) in six dimensions

• Didn’t we already know this?
  • We hinted at it before. This is our first time seeing that it would require an infinite number of corrections to cancel all the divergences.

• We call D the superficial degree of divergence, because there are exceptions to this rule.
  • If the loop momenta in the numerator cancel, we might get convergence even if \(D \geq 0\).
  • If \(D < 0\), we can still get divergence….
Divergences with $D < 0$

- Consider these diagrams:
  - Both the loop and the CT vertex are divergent, but these divergences cancel.
  - Even when the divergences don’t cancel, $Z$ factor can be adjusted to cancel these “divergent subdiagrams.”
  - In both cases, the theory is renormalizable despite the divergence.

- Thus, if all couplings have nonnegative mass dimensions, the theory is renormalizable.
  - Proving this is very difficult, so we won’t bother.
  - But, it turns out to be true for spin-0 and spin-1/2 fields.
  - It’s true for spin 1 fields when there is an associated gauge symmetry
  - It turns out that theories with spin > 1 are never renormalizable for $d \geq 4$. 