QFT

Chapter 17: Other One-Loop 1PI Vertices
N-Point Functions

- The exact vertex function is defined as:
  - “The sum of all 1PI diagrams with N external lines, and these N external propagators removed.”

- We considered this before for $n = 3$. Now let’s consider higher $n$ (still for $\phi^3$ theory).
  - No tree-level diagrams (obviously – the “point” vertex can by definition only attach to three things).
  - The one-loop contribution is finite for $d < 2n$. Since this is $\phi^3$ theory, we note that all one-loop diagrams are finite for $d = 6$. 
The 4-point 1PI vertex in $\varphi^3$ theory

- The result is that:

$$V_4 = \frac{g^4}{6(4\pi)^3} \int dF_4 \left( \frac{1}{D_{1234}} + \frac{1}{D_{1324}} + \frac{1}{D_{1243}} \right) + O(g^6)$$

$$D_{1234} = x_1x_4k_1^2 + x_2x_4k_2^2 + x_2x_3k_3^2 + x_1x_3k_4^2 + x_1x_2(k_1 + k_2)^2 + x_3x_4(k_2 + k_3)^2 + m^2$$
Conclusions

• All one-loop corrections will contribute to the vertex factor.

• These are finite and well-defined, so it is just a matter of drawing and calculating these vertices to the order at which we are working (each vertex introduces an additional factor of $g$).

• We now have the exact vertex up to one-loop. We’ve seen that the 3-point vertex requires us to redefine $Z_g$, the higher point vertices are already finite.