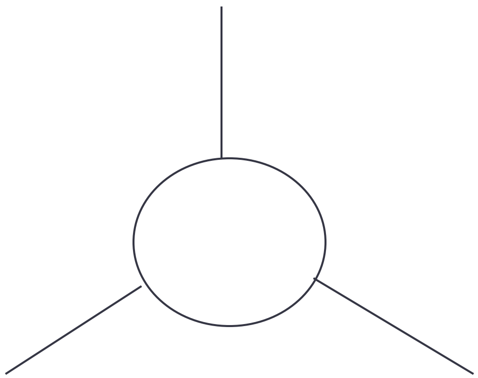
QFT

Chapter 16: Loop Corrections to the Vertex

Overview & Diagram

- In this chapter, we calculate the loop corrections to the vertex, just as we calculated the propagator corrections in chapter 14.

- The lowest-order vertex correction in φ^3 theory is:



Three-Point Vertex Function

- The exact vertex function is the sum of all 1PI diagrams that contribute, with all external lines incoming.

- For φ^3 theory, we have:

$$iV_3(k_1,k_2,k_3) = iZ_g g + (ig)^3\left(\frac{1}{i}\right)^3\int\frac{d^d\ell}{(2\pi)^d}\tilde{\Delta}((\ell-k_1)^2)\tilde{\Delta}((\ell+k_2)^2)\tilde{\Delta}(\ell^2) + O(g^5)$$

- We simplify this using the bag of tricks from chapter 14.

Boundary Condition of the Vertex Function

- The result is:

$$V_3(k_1, k_2, k_3) = g - \frac{1}{2} g\alpha \int dF_3 ln(D/m^2) - \kappa_C \alpha + O(\alpha^2)$$

- How do we fix k_c?
  - In the propagator, we got this by requiring the result to have the same pole as the (equivalent) Lehmann-Källér form.
  - For the vertex, there is nothing equivalent. Instead, we define:

$$V_3(0,0,0) = g$$

- There is no way to simplify this vertex factor.
  - We notice that for large k, the correction increases logarithmically with k. This is the same behavior we noted for the propagator.
  - We can also fix Z_g using the boundary condition.

A note on Mandelstam Variables

- Recall that $t = -(k_1 - k_{1'})^2$, $u = -(k_1 - k_{2'})^2$
- It would be nice to use this notation for vertex corrections, but there are no outgoing particles.
  - So, we assign $k_{1'} = -k_3$, $k_{2'} = -k_4$.
  - Then, $t = -(k_1 + k_3)^2$, $u = -(k_1 + k_4)^2$