QFT

Unit 12: Dimensional Analysis with ħ = c = 1

Overview

- This is a good chance to pause and review where we are.
- Then, in this very short section, we'll turn to dimensional analysis, and see what we can learn about various potential field theories on that ground.
  - We'll also evaluate φ^3 theory on these grounds.
- Next time, we'll take one more aside to talk about the "exact propagator" before turning to our next major topic: loop corrections.

Where are we?

- We saw that there was a problem with quantum mechanics – it's not relativistic. We made a few efforts to fix it, such as the Klein-Gordon equation and the Dirac Equation, but nothing really worked.
- We decided that the problem was with the way we treated position (but not time) as an operator, and decided to treat neither as an operator.
- Under this paradigm, we canonically quantized the field for spin-0 particles, along the way proving the Spin-Statistics Theorem.

Where are we?

- Next, we came up with the LSZ formula for scattering amplitudes, but that required being able to evaluate correlation functions.

- We spent 4 sections talking about path integrals, finally showing that:
  - We can take derivatives from Z(J), the ground state to ground state transition amplitude, to solve the correlation functions.
  - Z(J) can be easily expanded perturbatively in terms of Feynman Diagrams.

Where are we?

- We then derived the Feynman Rules so that we could calculate transition amplitudes directly from the Feynman diagrams.
- Finally, we converted the transition amplitudes into cross sections and decay rates, which can be experimentally measured.

Dimensional Analysis

- Thanks to our decision to set $\hbar = c = 1$, any given unit can be converted to mass units.
  - For example $[m] = 1$, since a mass already has mass units.
  - $[x] = -1$, since x is in distance units (perhaps meters), and $x\hbar^{-1}c$ is in mass^{-1} units.
  - Similarly, derivatives have mass units.

Dimensional Analysis, cntd.

- What about more interesting things?
  - Action appears in the exponential of the path integral, so it must be dimensionless.
  - This means that the Lagrangian must have dimension d.
  - This means that φ must have dimension .5(d-2)
  - This means that coupling constants must have dimension d - .5n(d-2), where n is the number of fields joined at a vertex.

Dimensional Analysis, cntd.

- This means that in φ^3 theory, the dimensionality of the coupling constant is .5(6-d).

- It turns out that we like theories with dimensionless coupling constants.
  - If not dimensionless, then the expansion will have to be in terms of $g*m^{-[g]}$ or $g*s^{-[g]/2}$ in order to have a dimensionless expansion
  - If [g] is negative, then the perturbative expansion breaks down, since later terms are more important than earlier terms. This means the theory is *nonrenormalizable.*
  - If [g] is positive, the theory becomes trivial at high energy, as any terms beyond the first vanish, and there is no dependence on g.

Dimensional Analysis, cntd.

- So, we'll continue our study of φ^3 theory in six dimensions. When we eventually construct the standard model, we'll have to check to make sure that the coupling constants are dimensionless in only four dimensions.
- It is for this reason that some books use φ^4 theory (see prbm. 9.2) as their toy model – that theory has a dimensionless coupling constant in four dimensions.
  - On the other hand, the diagrams are more complicated, since there are more particles!