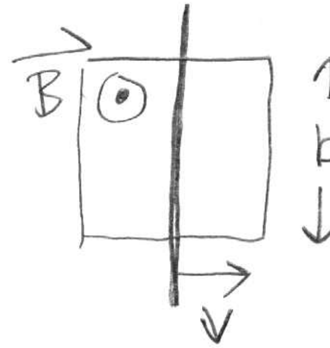
1. Look close up at the gap:



$$\mathcal{E} = \frac{1}{c}\frac{d\Phi_B}{dt}$$

$$= \frac{1}{c}\, bBv$$

$\underline{v}$: $x = x_0 \cos(\omega t)$

$$\omega = 2\pi\nu$$

$$\nu = 10^3 \text{ Hz}$$

$$\dot{x} = -\omega x_0 \sin(\omega t)$$

$$v = \text{maximum } \dot{x} = \omega x_0 = \underline{2\pi\nu x_0}$$

$$\boxed{\mathcal{E} = \frac{2\pi bB\nu x_0}{c}}$$

$b = 1$ cm $\quad B = 10^4$ G $\quad \nu = 10^3$ Hz $\quad x_0 = 10^{-1}$ cm

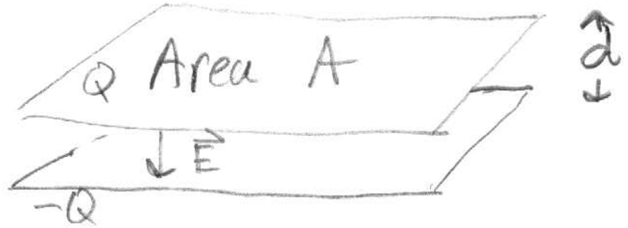
$c = 3 \cdot 10^{10}$ cm/s

$$\mathcal{E} = \frac{2 \cdot \pi \cdot 1 \cdot 10^4 \cdot 10^3 \cdot 10^{-1}}{3 \cdot 10^{10}}$$

$$= \frac{2\pi}{3} \cdot 10^{-4} \text{ statvolts}$$

$$\boxed{\begin{aligned} \mathcal{E} &= 2.1 \cdot 10^{-4} \text{ statvolts} \\ &= (2.1 \cdot 300) \cdot 10^{-4} \text{ volts} \\ &= 63 \text{ millivolts} \end{aligned}}$$

2. $\underline{C_0}$:



$\underline{\text{Recall}}$: $E = 4\pi\sigma = \dfrac{4\pi Q}{A}$

$$V = E \cdot d = \frac{4\pi d}{A} Q$$

$$Q = C_0 V = \left(\frac{A}{4\pi d}\right) V \qquad \boxed{C_0 = \frac{A}{4\pi d}}$$

$d/2 \rightarrow E = 4\pi Q/A$

$d/2 \rightarrow E = \dfrac{4\pi Q}{A\,\varepsilon}$

$\underline{\text{now}}$: $V = \dfrac{4\pi Q}{A}\dfrac{d}{2} + \dfrac{4\pi Q}{A\varepsilon}\dfrac{d}{2}$

$$Q = \frac{A}{4\pi d}\,\frac{2}{1+\frac{1}{\varepsilon}}\,V$$

$$\boxed{C = \frac{2}{1+\frac{1}{\varepsilon}}\,C_0}$$

Q_1 on $A/2$ $\quad Q_2$ on $A/2$ $\quad V = E_1 d = E_2 d$ (same voltage)

$E_1 = \dfrac{4\pi Q_1}{\varepsilon(A/2)}$ $\quad E_2 = \dfrac{4\pi Q_2}{(A/2)}$ $\Big\}\, d$ $\quad V = \dfrac{8\pi Q_1}{\varepsilon A} d = \dfrac{8\pi Q_2}{A} d$

$-Q_1 \quad Q_2$

$$Q_1 = \frac{\varepsilon A V}{8\pi d} \qquad Q_2 = \frac{A V}{8\pi d}$$

$$Q = Q_1 + Q_2 = \frac{A}{4\pi d}\,\frac{\varepsilon+1}{2}\,V$$

$$\boxed{C = \frac{1+\varepsilon}{2}\,C_0}$$

3. $E = \sqrt{(mc^2)^2 + (cp)^2}$

$$\underline{E} \cong cp\left[1 + \frac{m^2c^4}{2c^2p^2}\right]$$

$$\simeq cp + \frac{m^2c^3}{p}$$

$$\hbar\omega = c\hbar k + \frac{m^2c^3}{\hbar k}$$

$$\omega = ck + \frac{m^2c^3}{\hbar^2 k}$$

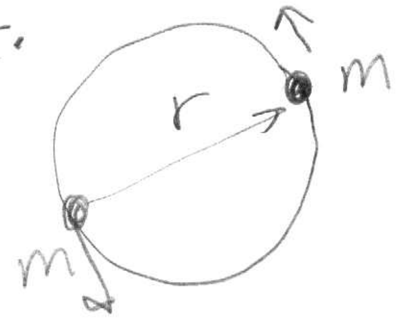
(a) Phase velocity

$$\boxed{\frac{\omega}{k} = c + \frac{m^2c^3}{(\hbar k)^2} = c\left(1 + \frac{(mc^2)^2}{(cp)^2}\right) > c}$$

(b) Group velocity

$$\boxed{\frac{d\omega}{dk} = c - \frac{m^2c^2}{(\hbar k)^2} = c\left(1 - \frac{(mc^2)^2}{(cp)^2}\right) < c}$$

4.



$$\mu \frac{v^2}{r} = G\frac{m^2}{r^2}$$

$$\frac{mv^2}{2r} = G\frac{m^2}{r^2}$$

$$\mu v r = \hbar$$

$$\frac{m}{2} v r = \hbar$$

$$v = \frac{2\hbar}{mr}$$

$$\frac{m}{2r} \cdot \left(\frac{4\hbar^2}{m^2 r^2}\right) = G\frac{m^2}{r^2}$$

$$\boxed{\frac{2\hbar^2}{m^3 \cdot G} = r} \quad (a)$$

$$\boxed{v = \frac{\cancel{2\hbar}}{m} \cdot \frac{m^3 G}{\cancel{2\hbar^2}\hbar} = \frac{m^2 G}{\hbar}}$$

$$E = \frac{1}{2}\mu v^2 - G\frac{m^2}{r}$$

$$= \frac{1}{2} \cdot \frac{m}{2} \cdot \frac{m^4 G^2}{\hbar^2} - \frac{Gm^2 \cdot m^3 \cdot G}{2\hbar^2}$$

$$\boxed{E = -\frac{1}{4}\frac{m^5 G^2}{\hbar^2}} \quad (b)$$

$$E < -2mc^2$$

$$-\frac{1}{4}\frac{m^5 G^2}{\hbar^2} < -2mc^2$$

$$m^4 > \frac{8\hbar^2 c^2}{G^2}$$

$$\boxed{m > 8^{1/4}\left(\frac{\hbar c}{G}\right)^{1/2}} \quad (c)$$

1) $\hbar = 1{,}05 \cdot 10^{-34} \frac{m^2 \cdot kg}{s}$

$$G = 6{,}67 \cdot 10^{-11} \frac{m^3}{kg\ s^2}$$

$$c = 3{,}00 \cdot 10^8 \frac{m}{s}$$

$$\left(\frac{\hbar c}{G}\right)^{1/2} = \left(\frac{1{,}05 \cdot 10^{-34} \frac{m^2 kg}{s} \cdot 3{,}00 \cdot 10^8 \frac{m}{s}}{6{,}67 \cdot 10^{-11}}\right)^{1/2}$$

$$= 2{,}17 \cdot 10^{-8}\ kg$$

$$\boxed{m > 8^{1/4} \cdot \left(\frac{\hbar c}{G}\right)^{1/2} = 3{,}65 \cdot 10^{-8}\ kg} \quad (d)$$

$m_p = 1{,}6726 \cdot 10^{-27}\ kg$, $m_p c^2 = 0{,}93827\ GeV$

$$1\ kg = \frac{0{,}93827}{1{,}67226 \cdot 10^{-27}} = 5{,}61 \cdot 10^{26}\ GeV$$

$$\boxed{mc^2 > 2,05 \cdot 10^{19}\ \text{GeV}} \quad (d)$$

2) $mc^2 > 8^{1/4} \left(\frac{\hbar c\ c^4}{G} \right)^{1/2}$

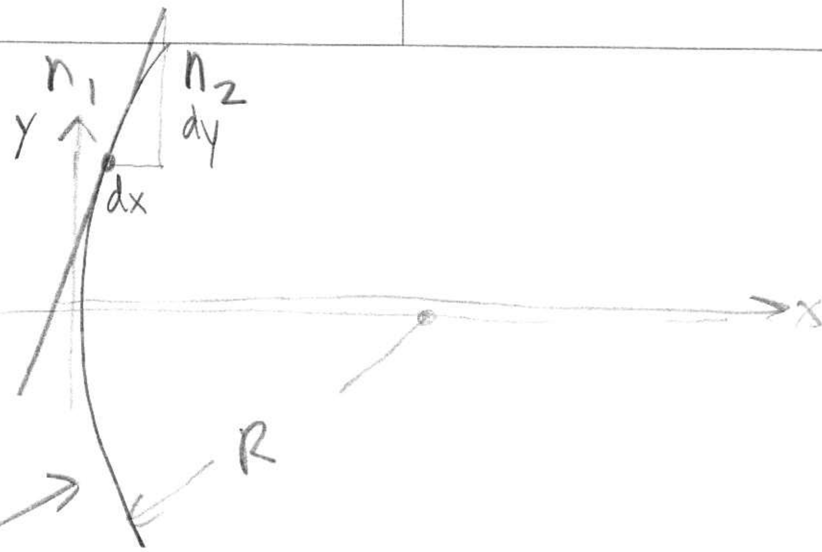
$$G = 6{,}709 \cdot 10^{-39}\ \hbar c\ \frac{c^4}{\text{GeV}^2}$$

$$\frac{G}{\hbar c\ c^4} = 6{,}709 \cdot 10^{-39}\ \frac{1}{\text{GeV}^2}$$

$$mc^2 > 8^{1/4} \cdot \left(\frac{1}{6{,}709 \cdot 10^{-39}} \right)^{1/2}$$

$$\boxed{mc^2 > 2.05 \cdot 10^{19}\ \text{GeV}} \quad (d)$$

5.



curve $(x-R)^2+y^2=R^2$

$$\cancel{x^2}-2xR+R^2+y^2=R^2$$

neglect

$$x=\frac{y^2}{2R}$$

$$\frac{dx}{dy}=\frac{y}{R}$$

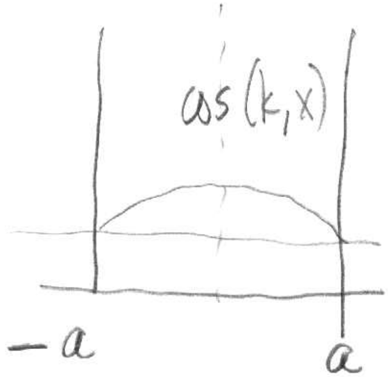
Snell: $n_2\theta = n_1\left(\frac{h}{p}+\frac{h}{R}\right)$

$$\frac{\cancel{h}}{q}=\frac{h}{R}-\theta=\frac{\cancel{h}}{R}-\frac{n_1}{n_2}\left(\frac{\cancel{h}}{p}+\frac{\cancel{h}}{R}\right)$$

so $$\boxed{\frac{n_1}{p}+\frac{n_2}{q}=\frac{(n_2-n_1)}{R}=\frac{1}{f}}$$ A

6. (a)

Ground state.



$$k_1 a = \frac{\pi}{2}$$

$$k_1 = \frac{\pi}{2a} = \frac{\pi}{2 \cdot 10^{-11}\ m}$$

$$\boxed{k_1 = 1.57 \cdot 10^{11}\ 1/m}$$

$$E = \frac{\hbar^2 k_1^2}{2m} = \frac{(\hbar c)^2 k_1^2}{2 m c^2}$$

$$= \frac{(197.3\ \text{MeV} \cdot \text{fm})^2 k_1^2}{2 \cdot 938}$$

put k_1 in $\frac{1}{\text{fm}}$...

$$a = 10^{-11}\ m$$

$$a = 10^{4}\ \text{fm}$$

$$k_1 = \frac{\pi}{2 \cdot 10^4}\ 1/\text{fm} = 1.57 \cdot 10^{-4}\ 1/\text{fm}$$

$$E_1 = \frac{197.3^2 \cdot \text{MeV}^2 \cdot \text{fm}^2 \cdot (1.57 \cdot 10^{-4})^2 \frac{1}{\text{fm}^2}}{2 \cdot 938\ \text{MeV}}$$

$$\boxed{\begin{aligned} E_1 &= 5.12 \cdot 10^{-7}\ \text{MeV} \\ &= 0.51\ \text{eV} \end{aligned}}$$

(b)

$$k_2 a = \pi$$

$$\boxed{k_2 = \frac{\pi}{a} = 2k_1 = \pi \cdot 10^{11} m = \pi \cdot 10^{-4} fm}$$

$$\boxed{E_2 = 4E_1 = 2.1\ \text{eV}}$$

(c) (i) $P(x,0) = |\Psi(x,0)|^2$

$$= \left|\frac{1}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}\psi_2(x)\right|^2$$

$$= \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2 + \frac{1}{2}\underbrace{\left[\psi_1(x)\psi_2^*(x) + \psi_1^*(x)\psi_2(x)\right]}_{2\,Re(\psi_1(x)\psi_2^*(x))}$$

$$\boxed{P(x,0) = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2 + Re\left[\psi_1(x)\psi_2^*(x)\right]}$$

(ii) $P(x,t) = |\Psi(x,t)|^2$

$$= \left|\frac{1}{\sqrt{2}}\psi_1(x)e^{\frac{-iE_1}{\hbar}t} + \frac{1}{\sqrt{2}}\psi_2(x)e^{\frac{-iE_2}{\hbar}t}\right|^2$$

$$\boxed{P(x,t) = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2 + \underbrace{Re\left[\psi_1(x)\psi_2^*(x)\,e^{\frac{i}{\hbar}(E_2-E_1)t}\right]}_{\text{repeats when}}}$$

(iii) $\frac{i(E_2-E_1)T}{\hbar} = 2\pi i$

$$T = \frac{2\pi\hbar}{(E_2-E_1)} = \frac{2\pi\hbar c}{(E_2-E_1)\cdot c}\frac{1}{c}$$

$$= \frac{2\pi \cdot 197.3\ eV\cdot nm}{(2.1-0.51)\ eV} \cdot \frac{1}{\underbrace{3\cdot 10^{10}}_{c\ m/s}\cdot\underbrace{10^{9}}_{nm/m}\ nm/s}$$

$$\boxed{T = 2.7\cdot 10^{-11}\ s}$$

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