Generalize the EM Wave

① Arbitrary Wavelength

$$E_0 \hat{x} \cos(y - ct) \rightarrow E_0 \hat{x} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right]$$

aka "wave number" k, $1/\ell$

$$= E_0 \hat{x} \cos(ky - ckt)$$

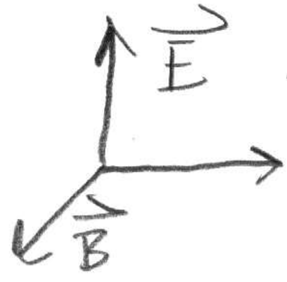
$\omega = ck$ or $c = \frac{\omega}{k}$

$$= E_0 \hat{x} \cos(ky - \omega t)$$

② Arbitrary direction!

$$ky \rightarrow \underbrace{(k_x \hat{x} + k_y \hat{y} + k_z \hat{z})}_{\text{wave propagates in this direction}} \cdot \underbrace{(x\hat{x} + y\hat{y} + z\hat{z})}_{\text{independent variable } \vec{r}}$$

$$ky \rightarrow \vec{k} \cdot \vec{r}$$



$\vec{k}$ (magnitude? $|\vec{k}| = \frac{2\pi}{\lambda}$)
propagation

note in free space

ⓐ $\vec{E} \times \vec{B}$ is $\parallel$ to $\vec{k}$

ⓑ $\vec{E}, \vec{B}$ have no components in direction of $\vec{k}$... $\vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{B} = 0$

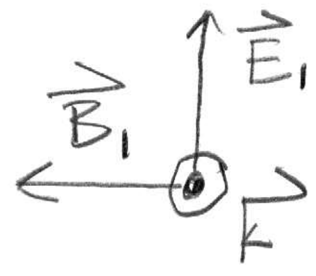
(c) $\vec{E} \rightarrow \vec{E}_\perp \cos(\vec{k}\cdot\vec{r} - \omega t + \alpha)$

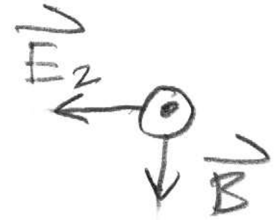
$\perp$ to $\hat{k}$ — arbitrary real phase!

2 possibilities!

(d) $\mathrm{Re}\left[\vec{E}_\perp e^{i(\vec{k}\cdot\vec{r} - \omega t + \alpha)}\right]$

(3) Look at wave coming at you...

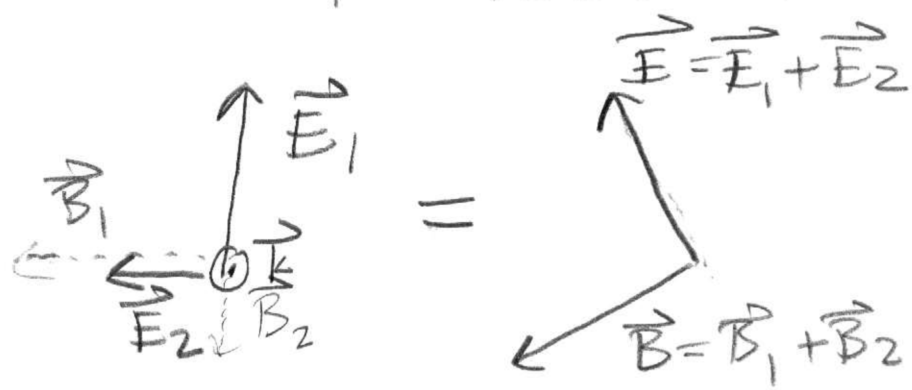




One dimension $\perp$ to $\vec{k}$

Second dimension $\perp$ to $\vec{k}$

Can linearly soperpose these two solutions... size of $\vec{E}_1$ and $\vec{E}_2$ are arbitrary $(\alpha_1 = \alpha_2)$



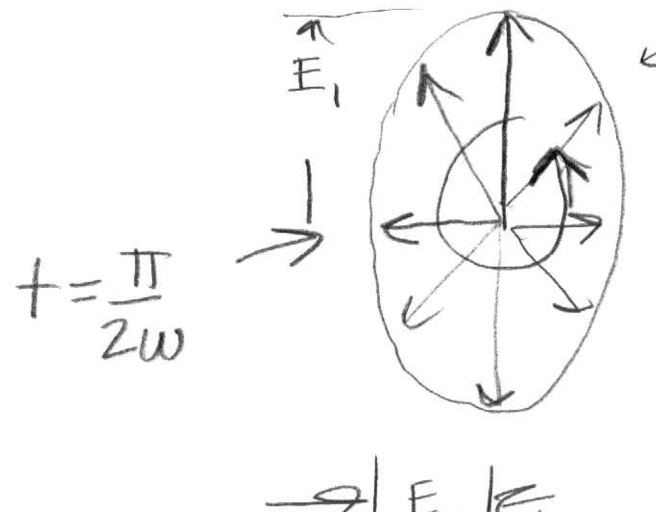
Linear Polarization

④ The Big Fun... $\alpha_1 \neq \alpha_2$

$$E_1 \propto e^{i(\vec{k}\cdot\vec{r}-\omega t)} \qquad E_2 \propto e^{i(\vec{k}\cdot\vec{r}-\omega t-\alpha_2)}$$

set $\alpha_1 = 0$ $\quad$ $\perp$ to E_1 $\quad$ think, $\alpha_2 = \pi/2$

Then $E_1 \propto \cos(\vec{k}\cdot\vec{r}-\omega t)$ $\quad$ $E_2 \propto \sin(\vec{k}\cdot\vec{r}-\omega t)$



$t = 0$

$\alpha_2 = \pi/2$

$E_2 \neq E_1$

elliptically polarized light

when $E_1 = E_2$ ... "circularly" polarized

As shown, "left-handed" (OPTICS)

CONFUSING!

looks "right-handed"

How do you reverse handedness?

$\Rightarrow \alpha_2 = 3\pi/2$] α_2 arbitrary,

Energy:

Density $\left(\frac{\text{ergs}}{\text{cm}^3}\right)$

Instantaneous

$$= \frac{1}{8\pi}\left(\underbrace{E^2(t) + B^2(t)}_{\text{but } E = B\,!}\right)$$

$$= \frac{1}{4\pi} E^2(t)$$

$$\overline{\text{Density}} = \frac{1}{4\pi}\overline{E^2(t)} \Leftarrow \frac{1}{2}E_0^2$$

$$\overline{\text{Density}} = \frac{1}{8\pi}E_0^2$$

$$\text{Energy } \underline{\text{flux}} = \text{Density}\left(\frac{\text{ergs}}{\text{cm}^3}\right) \times c\left(\frac{\text{cm}}{\text{s}}\right)$$

$$S \Rightarrow \frac{\text{ergs}}{\text{cm}^2 \ \text{s}}$$

Power per cm^2

In MKS:

$$S = \frac{E_0^2}{377\,\Omega} \qquad \text{check units}$$

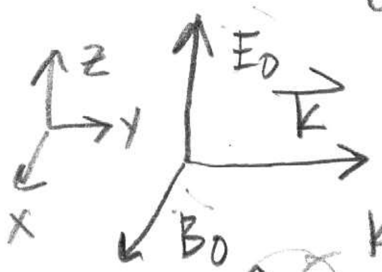
$$= \frac{(\text{volts}/\text{m})^2}{\Omega} = \frac{1}{\text{m}^2}\ \frac{\text{volts}^2}{\Omega}$$

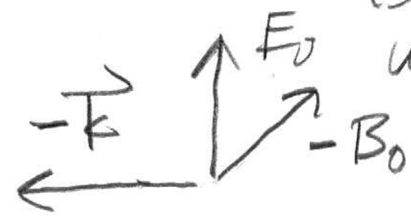
per unit area

like $\underbrace{\frac{V^2}{R}, IV, I^2R}_{\text{power}}$

Trapping Light Between Mirrors!

Or, radio waves





(standing wave!)

$k = \frac{2\pi}{\lambda}$

$$E_0 \hat{z} e^{i(\overbrace{\vec{k}\cdot\vec{r}}^{ky} - \omega t)}$$

$$B_0 \hat{x} e^{i(ky - \omega t)}$$

$$E_0 \hat{z} e^{i(-ky - \omega t)}$$

$$-B_0 \hat{x} e^{i(-ky - \omega t)}$$

!!

$$\vec{E}(t) = \mathrm{Re}\left[E_0 \hat{z} e^{i(ky - \omega t)} + E_0 \hat{z} e^{i(-ky - \omega t)}\right]$$

$$= E_0 \hat{z}\, \mathrm{Re}\left[\left(e^{iky} + e^{-iky}\right) e^{-i\omega t}\right]$$

$$\boxed{\vec{E}(t) = 2 E_0 \hat{z} \cos(ky) \cos(\omega t)}$$

$$\vec{B}(t) = \mathrm{Re}\left[B_0 \hat{x} e^{i(ky - \omega t)} - B_0 \hat{x} e^{i(-ky - \omega t)}\right]$$

$$= B_0 \hat{x}\, \mathrm{Re}\left[\underbrace{\left(e^{iky} - e^{-iky}\right)}_{2i \sin(ky)} \underbrace{e^{-i\omega t}}_{\times (\cos(\omega t) + i \sin(\omega t))}\right]$$

$$\boxed{\vec{B}(t) = -2 B_0 \hat{x} \sin(ky) \sin(\omega t)}$$

Standing Waves! Fun part is...
comparing E + B!