X-ray Diffraction

Photons can have very short wavelengths. Visible light... \( \lambda = 500\text{nm} \)
diffraction grating \( \equiv \) few \( \times \) spacing

X-rays... \( \lambda \approx 0.1\text{nm} \)
array of atoms separated by \( \approx \) few nanometres.

Complications: generally see effect in reflection

- details of how photons actually interact a bit involved... Just accept that recurring patterns of atoms ("unit cells") scatter x-rays in all directions; some directions see constructive interference.
- geometry a bit different because of non-1-d nature of the crystal.
- look at... different paths that end up going in same final direction get phase difference.
- look for a variety of "planes" of unit cells.
Note: θ is not with respect to vertical convention in this work.

Wave fronts along different rays all "start at the same line above.

1+2 hit the finish line after each has traveled $a_0 \cdot \cos \theta$. Difference in path length is $0!$, CENTRAL MAX.

1+3 \( \Rightarrow \) 3 goes EXTRA $2 \times a_0 \cdot \sin \theta$.

bigger max: $2a_0 \sin \theta = m \lambda$ \( m = \text{integer} \)

first occurrence of Bragg's Law!
Most other planes of atoms can do this!

\[ d = a_0 \sqrt{2} \]

These planes:

- \( d = a_0 \cos(\text{angle}) \)
- \( b = a_0 \frac{1}{\sqrt{1 + b^2}} \)
- \( c = 2a_0 \sin B = m \times \) Bragg

...extra distance

Look at plane crossings...
Polarization of Light + E&M Radiation

Linear: what we've learned so far

- 2 linear polarizations: "span the space," like vertical & horizontal (can tilt)
- $\vec{E} \times \vec{B} = \text{direction of motion}$

[Diagram showing a wave with X and Z axes, motion in Z, and waves labeled "behind" and "other"]

which is $\vec{E}$? Answer: vertical
B? Answer: horizontal
else motion OTHER WAY!

From behind other

- can make simple linear combo: for example
- can delay maximum of one, relative to other circular polarization.
Some wonderful animations on Wikipedia (Circular Polarization)
   Youtube (Linear, Circular, and Elliptical...)
   QOgrU4npr80

Polarization is like a "hidden variable" with light, EM radiation:
- Sunlight from blue sky is polarized
- Light that reflects off shiny surfaces is polarized \( \pi \) to surface
- Polaroid sunglasses use polarization
- Some birds, insects use polarization to navigate
- Radio & WiFi want to be unpolarized, better penetration past random obstacles...
  problem is, antennas tend to emit polarized light
- Polarization of starlight helps to reconstruct galactic magnetic fields
- 3-D glasses these days use circular polarization

Polarizing Sheets: block component of \( \overrightarrow{E} \parallel \) to their direction
   \( \perp \) transmit
\[ \begin{align*}
\text{Sheet} & \rightarrow \uparrow \quad \vec{E}_y \quad \vec{E}_x \quad \downarrow \\
2 \angle & \text{ lines denote polarization direction}
\end{align*} \]

Only \( \vec{E}_y \) penetrates, \( \vec{E}_x \) blocked.

\[ |\vec{E}_y| = |\vec{E}| \cos \Theta \]

\[ I = I_0 \cos^2 \Theta \]

Now imagine crossed polarizers.

Does any light penetrate for any \( \Theta \)? NO

What happens if a third polarizer is inserted at 45\(^\circ\), between?

\[ \Rightarrow \text{Light gets through!} \]
Take it in steps:

Step #1: Incident \( \vec{E} \) full strength

Step #2: Approach 45°

\[ \frac{E \cos \theta}{\sqrt{2}} \]

Step #3: Approach final horizontal

\[ \frac{E \cos \theta}{\sqrt{2}} \]

\[ \frac{1}{2} E \cos \theta \] penetrates

\[ I \propto \frac{1}{4} \text{Im} \cos^2 \theta \]
Polarization By Reflection
(Brewster's Angle)

Remember: \[ I \propto \sin^2 \theta \]

\[ \vec{E} \] (polarized)

How light propagates:

incident \[ \vec{E} \] \( \rightarrow \) \( \vec{e} \) \( \rightarrow \) \( \vec{r} \)

\( V = c \) \( \Rightarrow \) \( P \) causes \( \vec{e} \) to oscillate.

\[ V = \frac{c}{n} \]

Consider 2 polarizations:

\( \eta_1 = 1 \)

\( \eta_2 = n \)

Think, when 90° here all in plane of paper

all in plane of surface

\( \Theta_p \) \( \Theta_p \)

\( \beta_r \) \( \beta_r \)
\[ n \sin \theta_r = \sin \theta_p \]

\[ \theta_p + \theta_r = 90^\circ \]

\[ \theta_r = 90^\circ - \theta_p \]

\[ n \cdot \sin (90^\circ - \theta_p) = n \cos \theta_p = \sin \theta_p \]

\[ \frac{\tan \theta_p}{n} = \frac{1}{n} \]

\[ n = 1.5 \]

\[ \theta_p = 56.3^\circ \]

\[ \theta_r = 90^\circ - 56.3^\circ = 33.7^\circ \]

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Birefringence / Double Refraction

When molecules go wonky

2 polarizations in some directions have different speeds...

One axis

Extraordinary

angle of polarization \( e'' \)

\[ n_0 = n_e(\theta) \]

\[ v_c = \frac{c}{n_e} \]

\[ n_e < n_0 \]

Calculate:

\[ n_0 = 1.658 \]

\[ n_e = 1.486 \]

\[ v_0 = \frac{c}{n_0} \]

speed / ordinary ray / isotropic
spot \rightarrow Huygens

material

\text{optical axis}

\text{et al. together}

Optic axis

\text{I can delay one polarization?}

\text{(how to turn linear polarization into circular)}

Optic axis

SNELL VIOLATED

fus\rightarrow
Problem 18

$\theta_i = 38.8^\circ$

$\tan \theta_0 = \frac{x_0}{+}$

$\tan \theta_e = \frac{x_e}{+}$

$n$ larger

$n_e$ smaller

$n_0 = 1.658$

$n_e = 1.48$

$\sin \theta_i = n_0 \sin \theta_0$

$\sin \theta_i = n_e \sin \theta_e$

$\sin \theta_0 = \frac{\sin \theta_i}{n_0}$

$\sin \theta_e = \frac{\sin \theta_i}{n_e}$

$\sin \theta_0 = \frac{\sin \theta_i}{\sqrt{n_0^2 - \sin^2 \theta_i}}$

$\sin \theta_e = \frac{\sin \theta_i}{\sqrt{n_e^2 - \sin^2 \theta_i}}$

$\tan \theta_0 = \frac{\sin \theta_i}{\sqrt{n_0^2 - \sin^2 \theta_i}}$

$\tan \theta_e = \frac{\sin \theta_i}{\sqrt{n_e^2 - \sin^2 \theta_i}}$

$n_0 = 1.658$ (when circular shell respected)

$x_0 = \frac{+ \tan \theta_0}{+ \sin \theta_i}$

$x_e = \frac{+ \tan \theta_e}{+ \sin \theta_i}$

$x_0 = 0.457 \text{ cm}$

$x_e = 0.521 \text{ cm}$

$\text{distance} = x_e - x_0 = 0.064 \text{ cm}$
(b) o-ray has bigger index, bent more, so o-ray was \( \times \), e-ray was \( \checkmark \).

(c) would see a polarization if polarizer in the plane of page, e if \( \perp \) to plane of page.
Circular Polarization

Just draw $\vec{E}$ for 2 polarization

Vertical $y$

$x$

$E_y$

$E_x$

In phase

Imagine delaying by $\frac{1}{4}$ wavelength

$y$

$x$

$y$
x advanced by \( \frac{1}{4} \) wave

How short by a lot

\( \frac{1}{4} \) wave

\( \pm \sqrt{n_0} \) or \( \pm \sqrt{n_e} \)

\( = \frac{1}{4} \)

or \( (n_0 - n_e) \times = \frac{1}{4} \)