X-ray Diffraction

Photons can have very short wavelengths.

visible light... $\lambda = 500nm$

diffraction grating $\approx$ few λ spacing

X-rays... $\lambda \sim 0.1nm$

array of atoms separated by $\approx$ few nanometers.

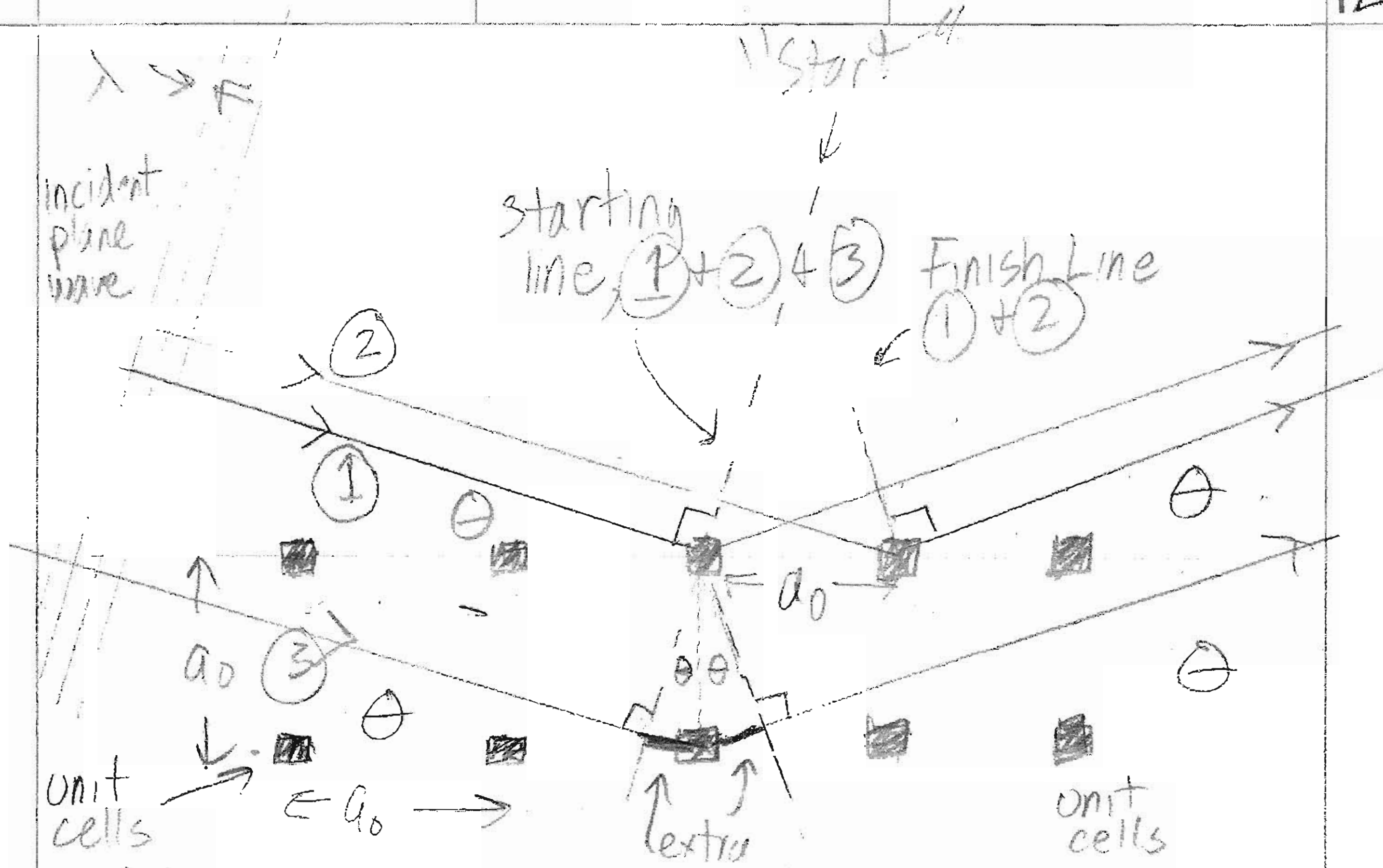
Complications: • generally see effect in reflection

• details of how photons actually interact a bit involved.. just accept that recurring patterns of atoms ("unit cells") scatter x-rays in all directions; some directions see constructive interference..

• geometry a bit different because of non 1-d nature of the crystal

• look at.. different paths that end up going in same final direction - get path difference

• look for a variety of "planes" of "unit cells"

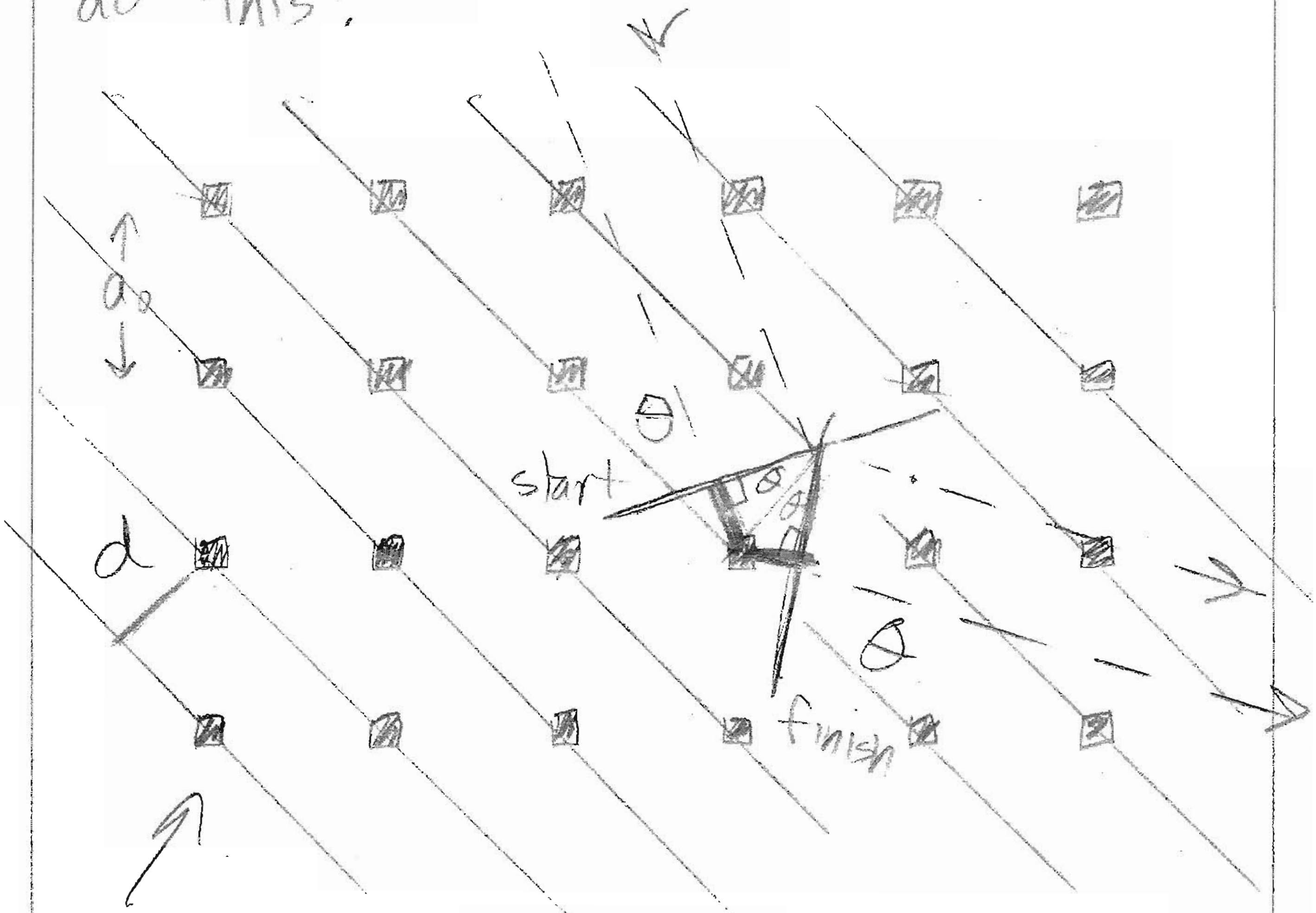


- Note: θ not with respect to vertical... convention in this work
- Wave fronts along different rays all "start" at the same line above.
- 1 + 2 hit there finish line after each has traveled $a_0 \cdot \cos\theta$ ... difference in path length is 0! CENTRAL MAX
- 1 + 3 ... 3 goes EXTRA $2 \times a_0 \cdot \sin\theta_0$

bigger max → $\boxed{2a_0 \sin\theta = m\lambda}$ m = integer

first occurrence of Bragg's Law!

Many other planes of atoms can do this!



these planes: $d = a_0/\sqrt{2}$

- more generally, 1 step down, b steps over (b=1 above)

$d = a_0 \cos(\text{angle})$ $\quad b \to \infty,\ d = a_0$

$d = a_0 \cdot \frac{b}{\sqrt{1+b^2}}$

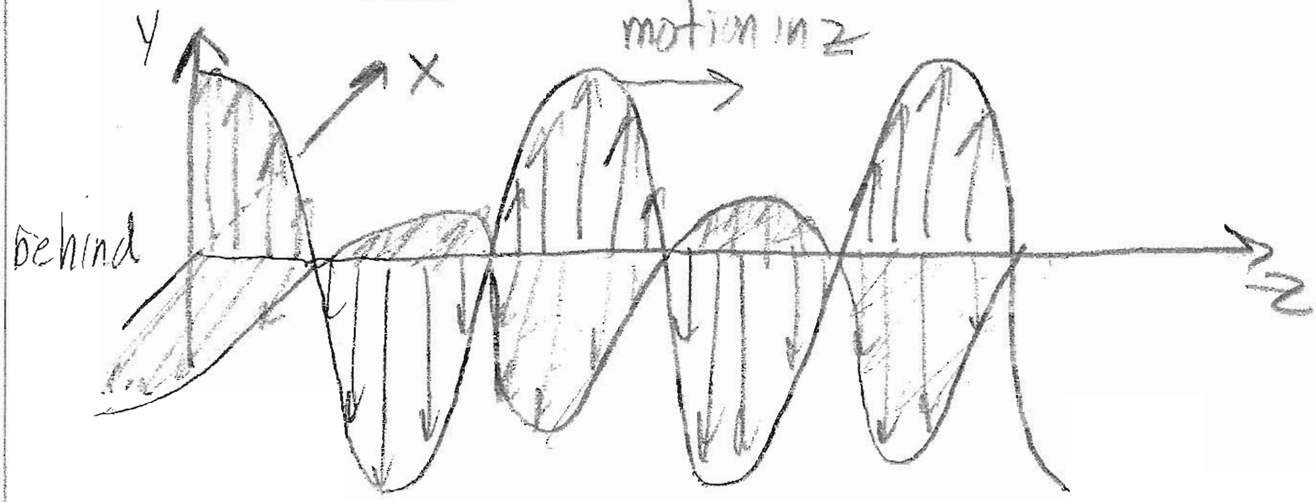
- look at plane crossings... extra distance $= 2d\sin\theta = m\lambda$. (Bragg).

Polarization of Light + EM Radiation

Linear: what we've learned so far

- 2 linear polarizations... "span the space", like vertical + horizontal (can tilt)

$\vec{E} \times \vec{B}$ = direction of motion

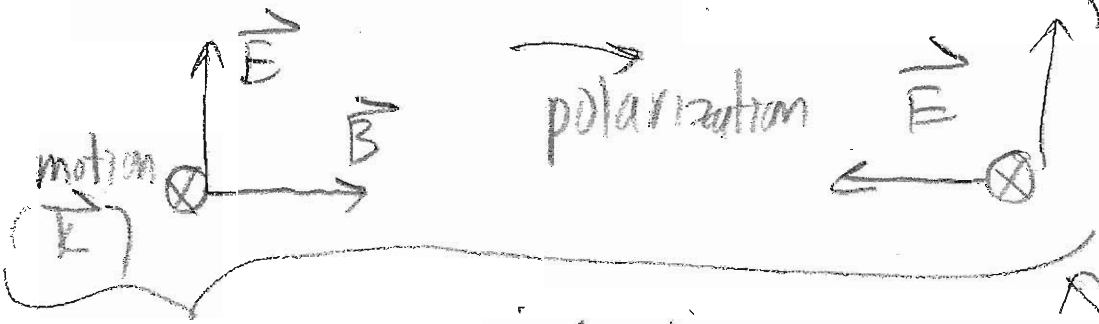


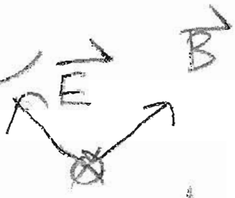
which is $\vec{E}$? Answer... vertical

$\vec{B}$? Answer... horizontal

else motion OTHER WAY!

From behind   other





- can make simple linear combo: for example
- can delay maximum of one, relative to other: circular polarization

Some wonderful animations on ...
Wikipedia (Circular Polarization)
Youtube (Linear, Circular, and Elliptical ...)
Q0qrU4npr80

Polarization is like a "hidden variable" with light, EM radiation

→ Sunlight from blue sky is polarized

→ Light that reflects of shiny surfaces is polarized ∥ to surface.

→ Polaroid Sunglasses. use polarization

→ Some birds, insects use polarization to navigate

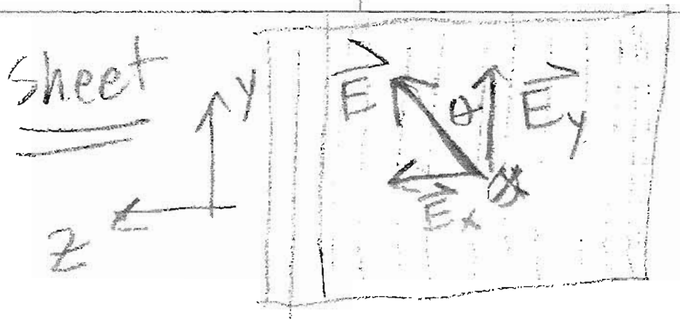
→ Radio + WiFi want to be unpolarized, better penetration past random obstacles...
problem is, antennas tend to emit polarized light

→ Polarization of starlight helps to reconstruct galactic magnetic fields

→ 3-D glasses these days use circular polarization

Polarizing Sheets: • block component of $\vec{E}$ ⊥ to their direction

• transmit ∥



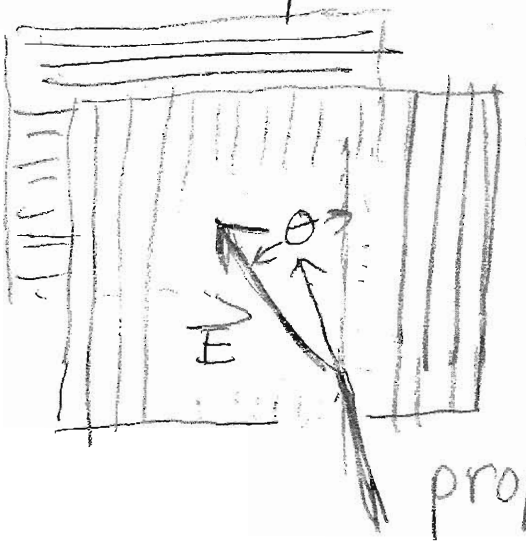
$\leftarrow \parallel$ lines denote polarization direction

only $\vec{E}_y$ penetrates, $\vec{E}_x$ blocked.

$$|\vec{E}_y| = |\vec{E}|\cos\theta$$

$$I = I_m \cos^2\theta$$

Now imagine crossed polarizers



propagating in

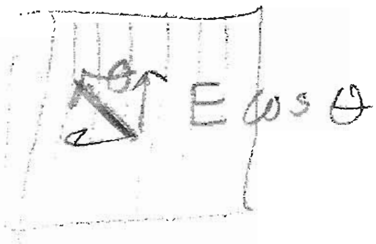
Does any light penetrate, for any θ? NO

What happens if... a third polarizer is inserted at 45°, between?

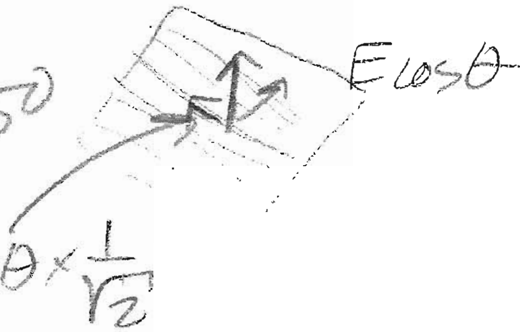
$\Rightarrow$ Light gets through!

Take it in steps:

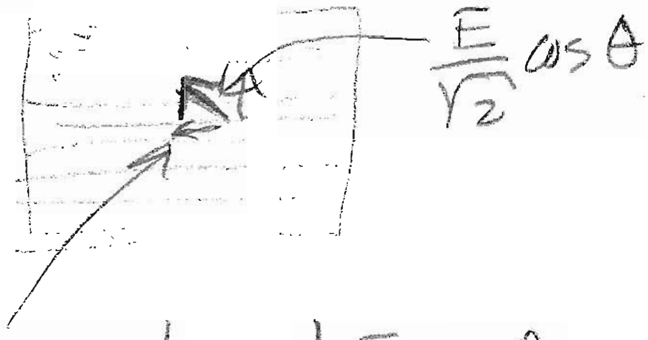
Step #1: Incident $\vec{E}$ full strength



Step #2: Approach 45°



Step #3 Approach final horizontal



$$\frac{E}{\sqrt{2}}\cos\theta \times \frac{1}{\sqrt{2}} = \frac{1}{2}E\cos\theta$$

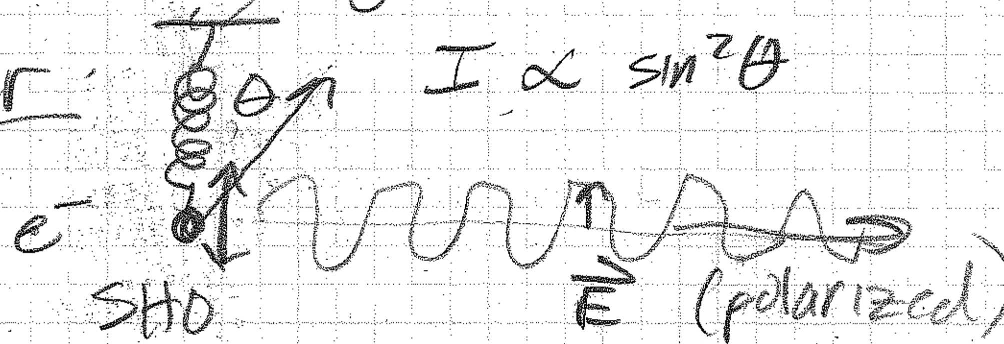
penetrates

$$I \propto \frac{1}{4} I_m \cos^2\theta$$

Polarization By Reflection

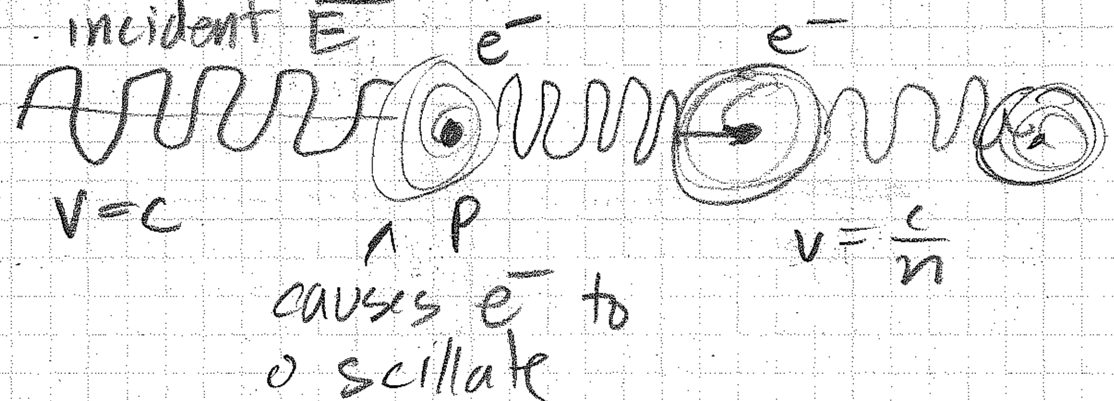
(Brewster's Angle)

Remember:

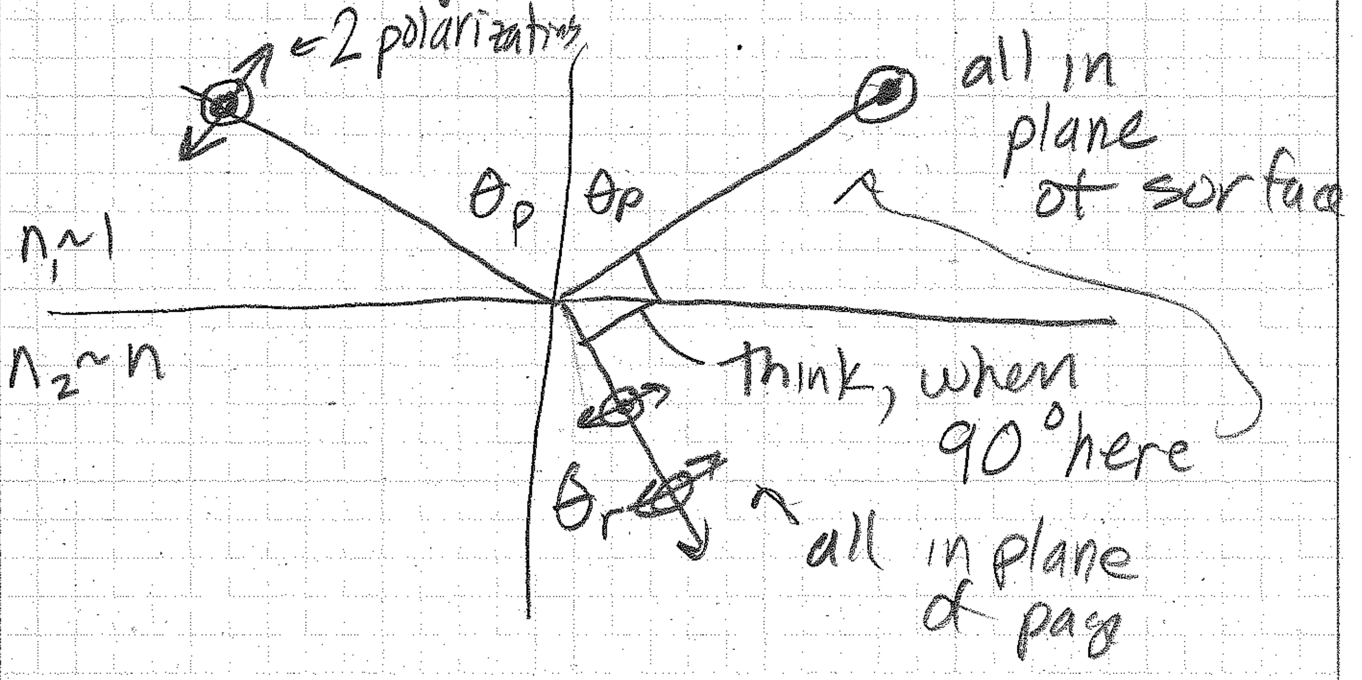


$I \propto \sin^2\theta$

How light propugates



Consider 2 polarizations



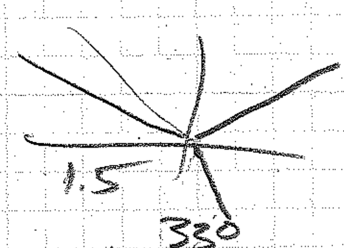
$$n \sin\theta_r = \sin\theta_p$$

$$\theta_p + \theta_r = 90^\circ$$

$$\theta_r = 90^\circ - \theta_p$$

$$n \sin(90^\circ - \theta_p) = n\cos\theta_p = \sin\theta_p$$

$$\boxed{\tan\theta_p = n}$$



$\underline{n = 1.5}$ $\quad \theta_p = 56.3^\circ$

$$\theta_r = 90 - 56.3$$
$$= 33.7^\circ$$

Birefringence / Double Refraction ..
when molecules go wonky

2 polarizations in some directions have different speeds...

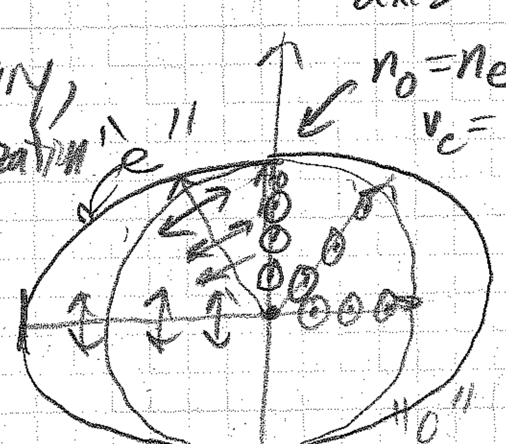
optic axis

$n_o = n_e(\theta)$ observe: $\vec{E} \parallel$ for all

$v_e = \frac{c}{n_e}$

extraordinary, "e"
angle of polarization changes

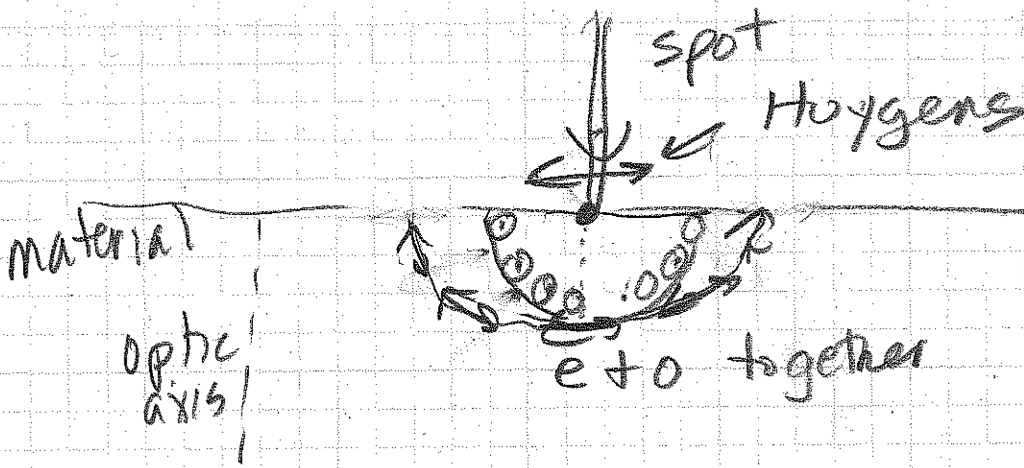
$n_e < n_o$

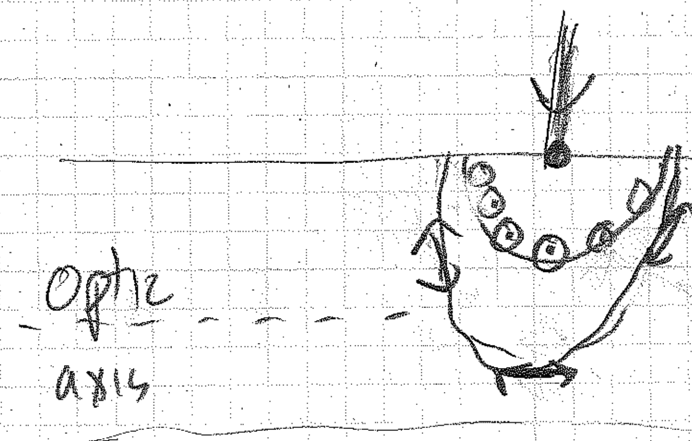


"o" $\quad v_o = \frac{c}{n_o}$

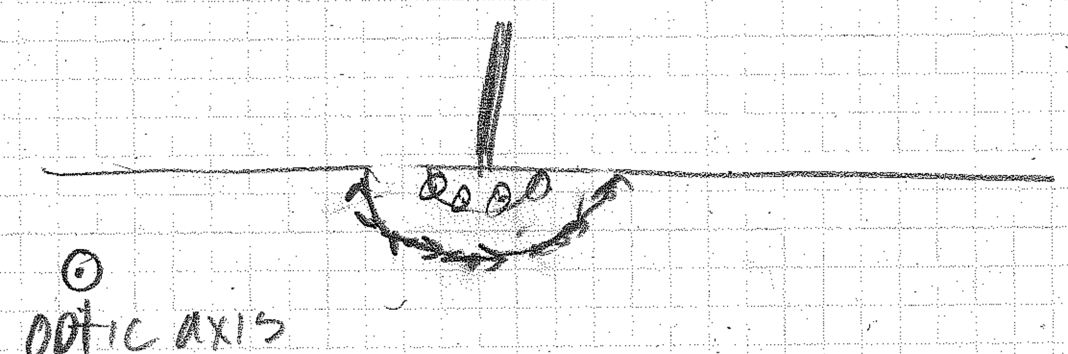
Calculate:
$n_o = 1.658$
$n_e = 1.486$

speed, ordinary ray isotropic

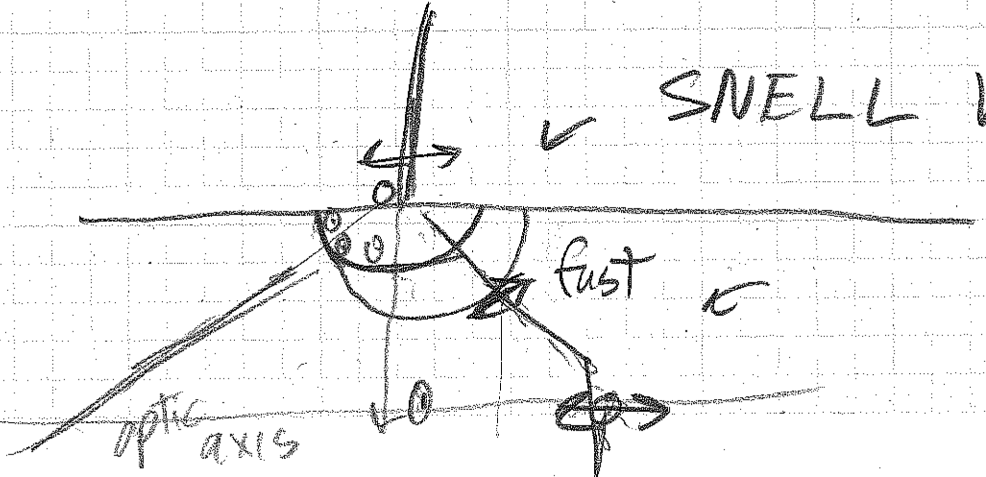




} can delay one polarization!!
(how to turn linear polarization into circular)



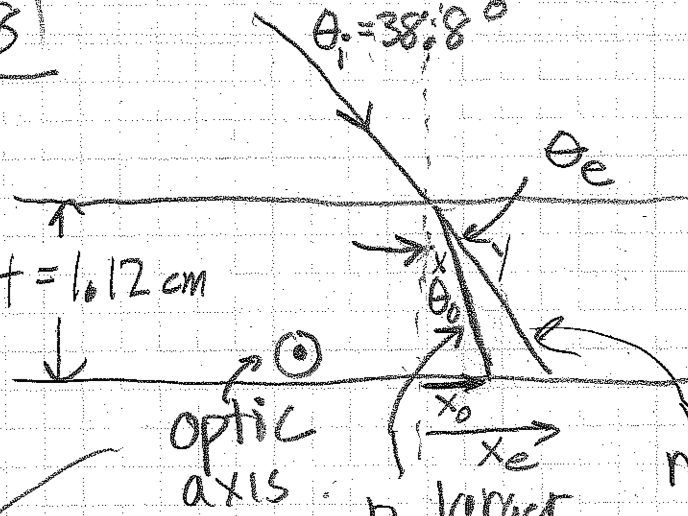
SNELL VIOLATED.



Chapter 48

Problem 18

(a)

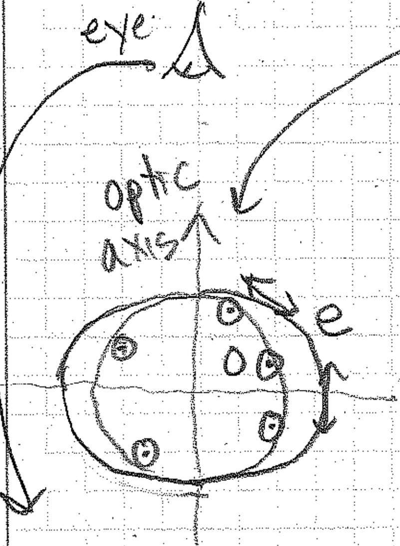


$\tan\theta_0 = \frac{x_0}{t}$

$\tan\theta_e = \frac{x_e}{t}$

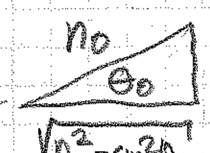
$n_e = 1.48$

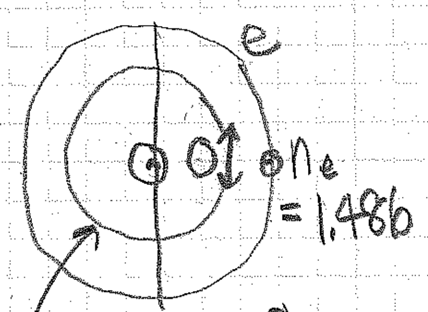
$n_0 = 1.658$



$\sin\theta_i = n_0 \sin\theta_0$ $\quad$ $\sin\theta_i = n_e \sin\theta_e$

$\sin\theta_0 = \frac{\sin\theta_i}{n_0}$ $\quad$ $\sin\theta_e = \frac{\sin\theta_i}{n_e}$





$n_0 = 1.658$

when circular, snell respected

$\tan\theta_0 = \frac{\sin\theta_i}{\sqrt{n_0^2 - \sin^2\theta_i}}$ $\quad$ $\tan\theta_e = \frac{\sin\theta_i}{\sqrt{n_e^2 - \sin^2\theta_i}}$

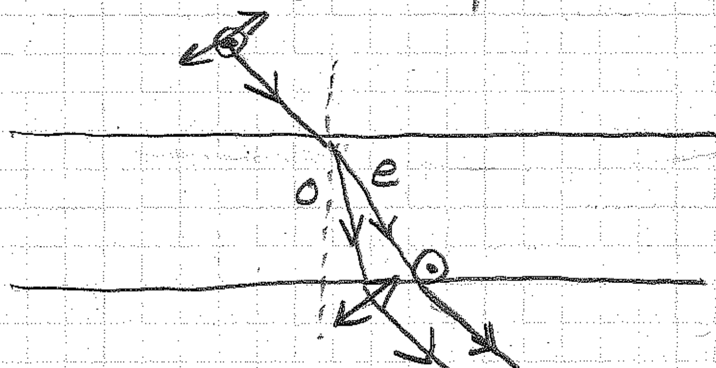
$x_0 = t\tan\theta_0 = \frac{t\sin\theta_i}{\sqrt{n_0^2 - \sin^2\theta_i}}$ $\quad$ $x_e = t\tan\theta_e = \frac{t\sin\theta_i}{\sqrt{n_e^2 - \sin^2\theta_i}}$

$x_0 = 0.457$ cm $\quad$ $x_e = 0.521$ cm

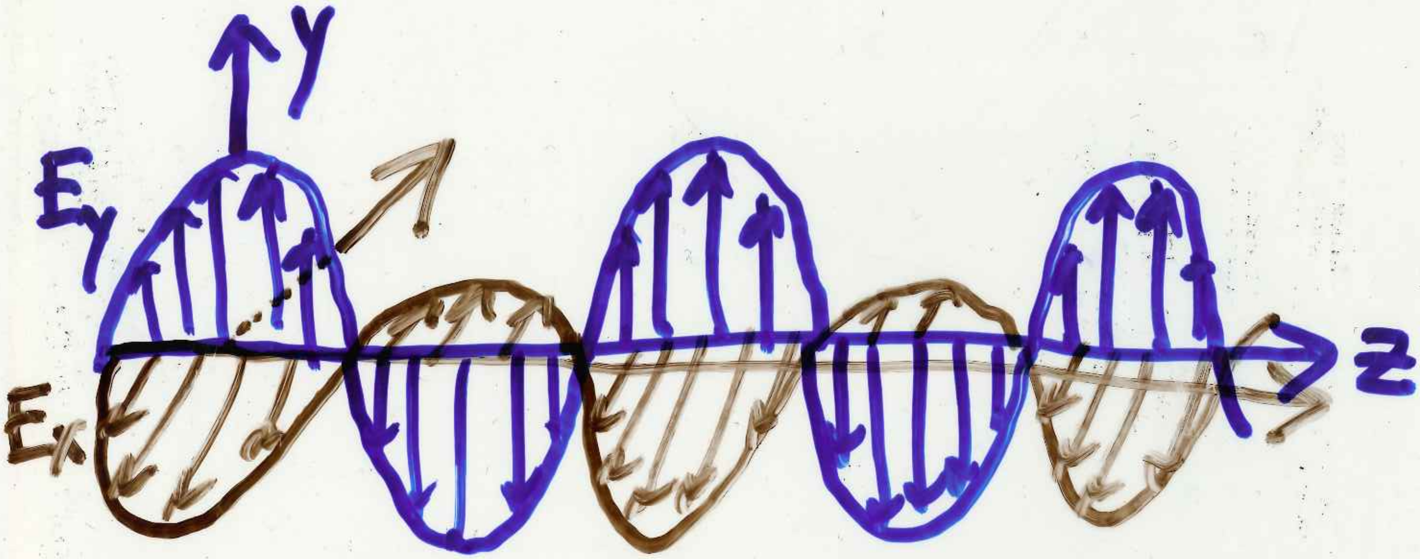
$$\boxed{\text{distance} = x_e - x_0 = 0.064 \text{ cm}}$$

(b) o-ray has bigger index, bent more, so,
o-ray was $\underline{X}$, e-ray was $\underline{Y}$

(c)



(d) would see o polarization if polarizer in the plane of page, e if $\perp$ to plane of page

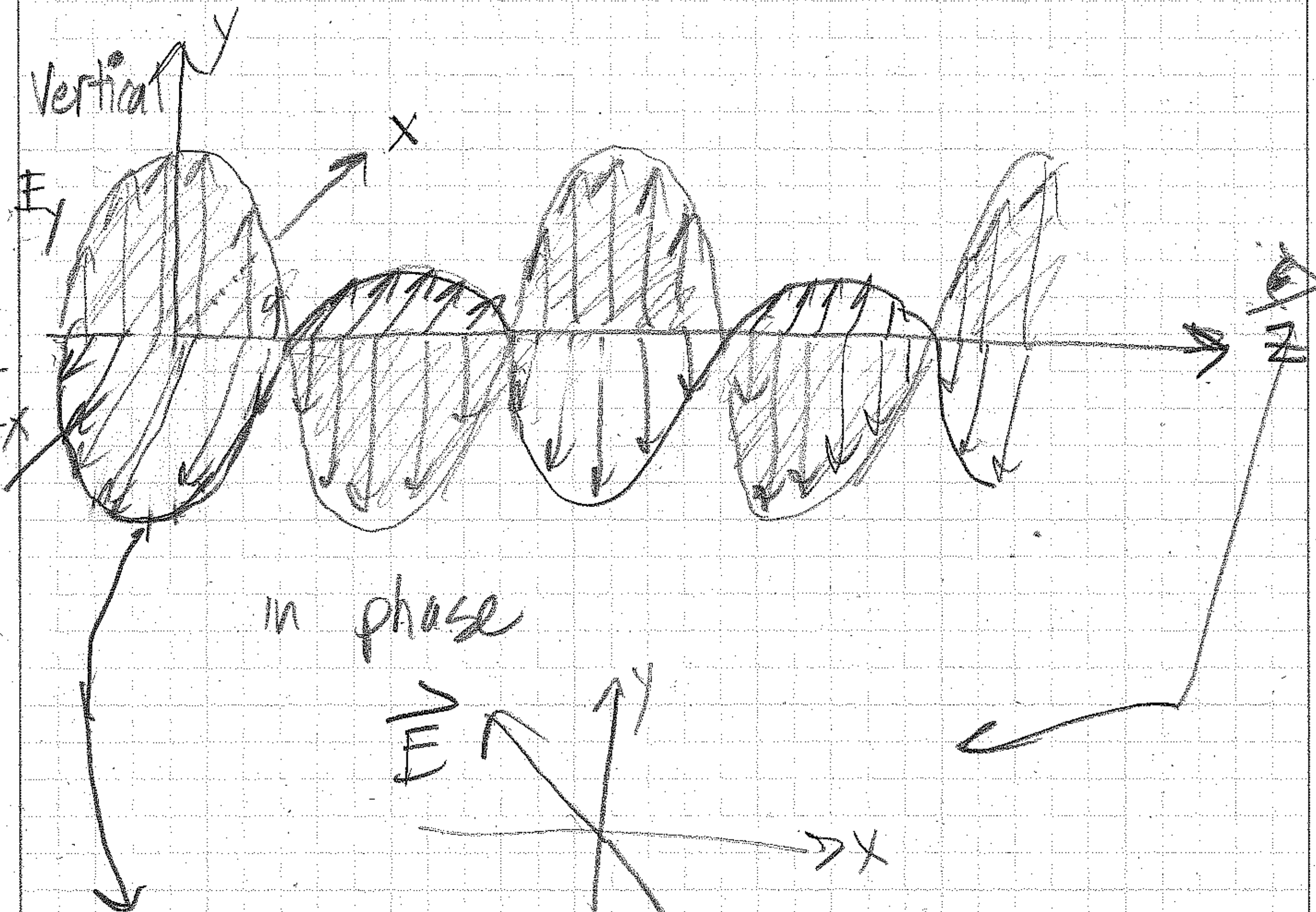


y
E_y
E_x
z

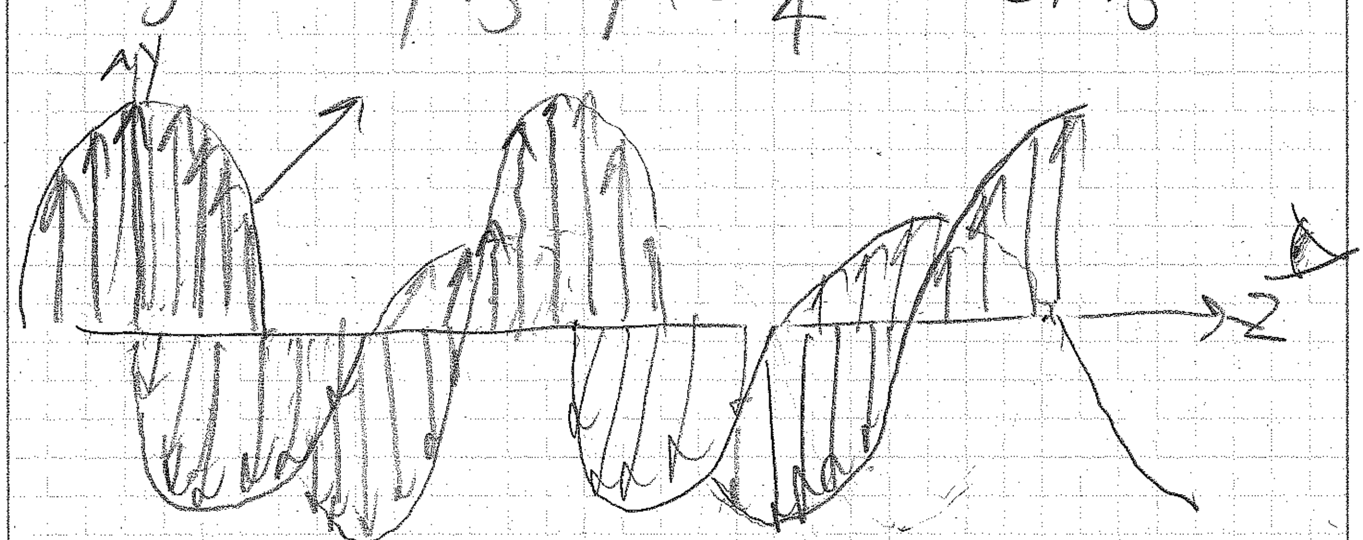
y
Ey
Ex
z

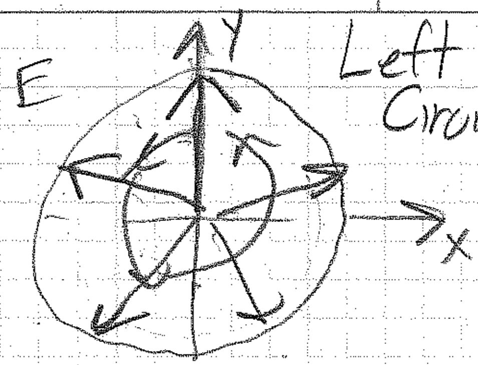
Circular Polarization

Just draw $\vec{E}$ for 2 polarization

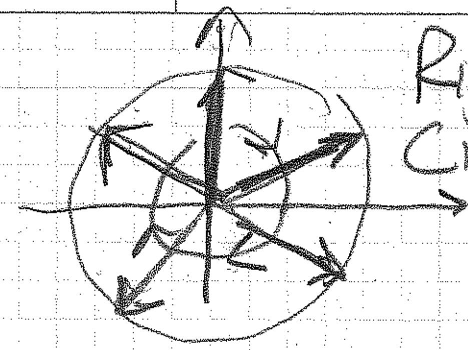


Imagine delaying by $\frac{1}{4}$ wavelength





Left Circular

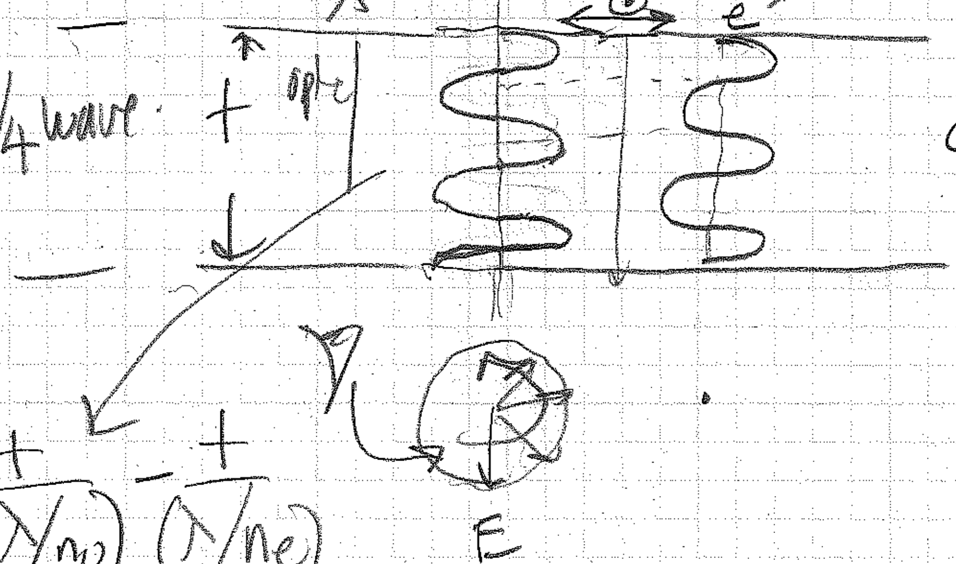


Right Circular

x advanced
by $\frac{1}{4}$ wave

$n_o > n_e$ by a lot

How ? o shorter λ ← o → e longer



$$\frac{t}{(\lambda/n_o)} - \frac{t}{(\lambda/n_e)}$$

$$= \frac{1}{4}$$

or $(n_o - n_e)\frac{t}{\lambda} = \frac{1}{4}$