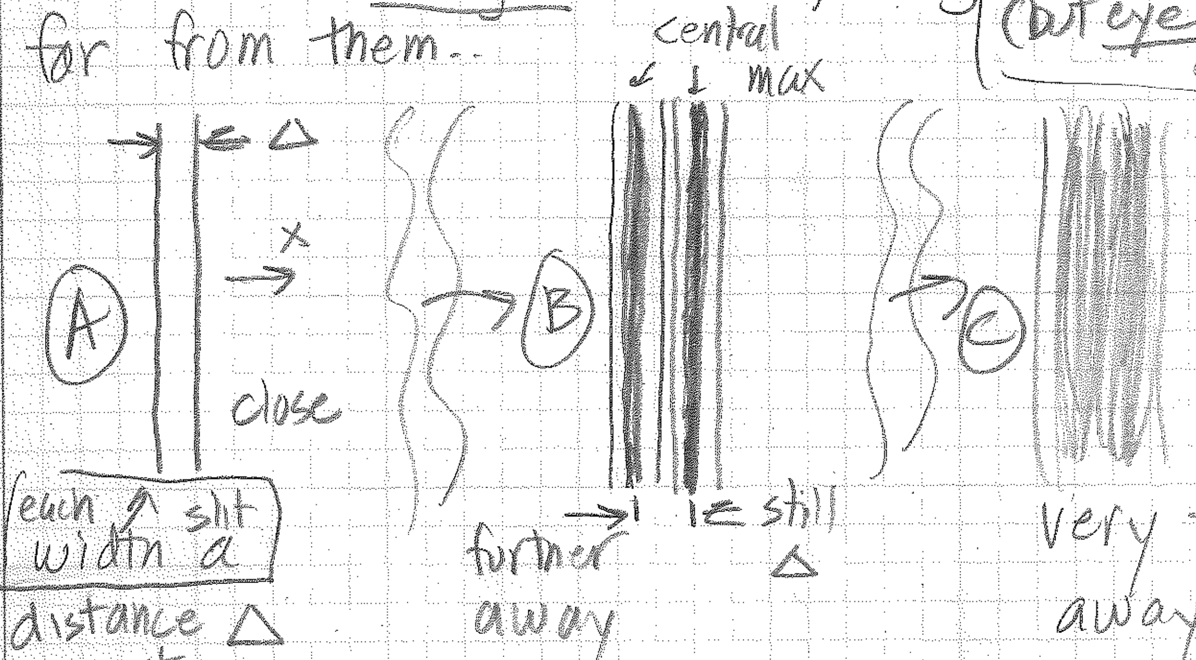
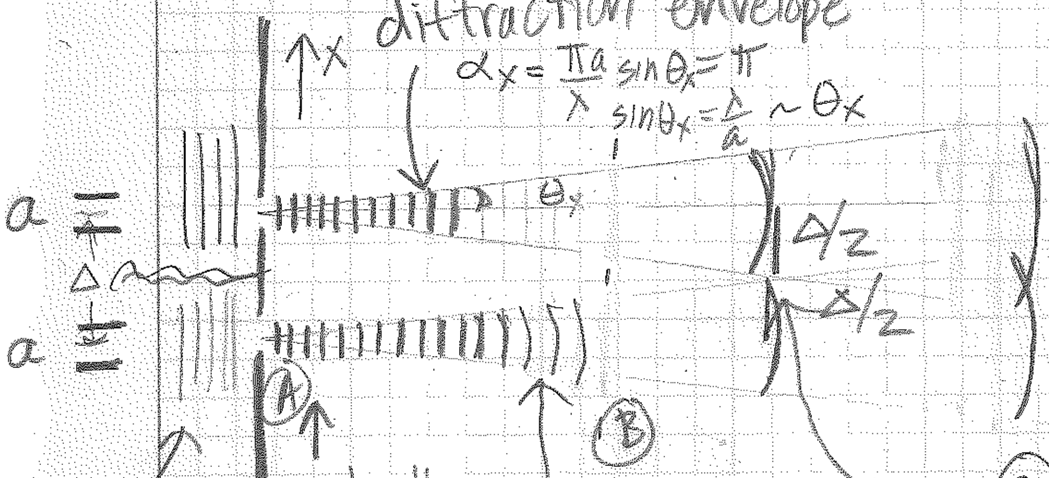
Merging Due to Diffraction (light merges, but angle distinct)

Two lines merge when you get too far from them.. (but eye still can resolve)



Assume light incoherent

Side view



2 beams NOT COHERENT

initially, each a wide

beam spreads out due to diffraction

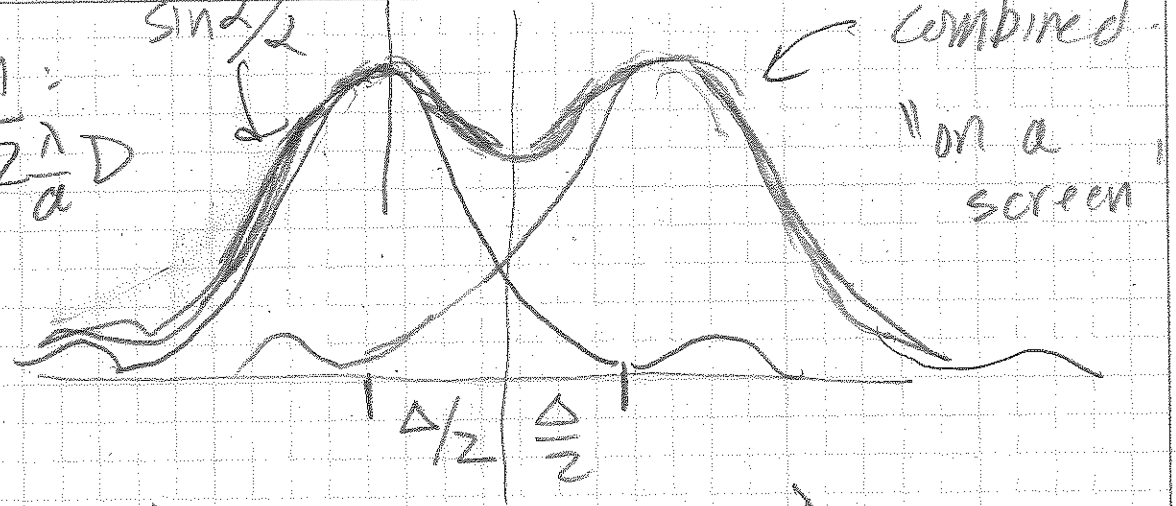
light overlaps in space, but light from different slits has different directions

$$\frac{\Delta}{2} \sim \theta_x D \sim \frac{\lambda}{a} D$$

$\leftarrow$ D $\longrightarrow$

Lines begin to merge $\Delta \lesssim 2\frac{\lambda}{a} D$

then: $\Delta \simeq 2\frac{\lambda}{a} D$



Complete merging: $\Delta \sim \frac{\lambda}{a} D$

or $D \sim \frac{a}{\lambda} \Delta$

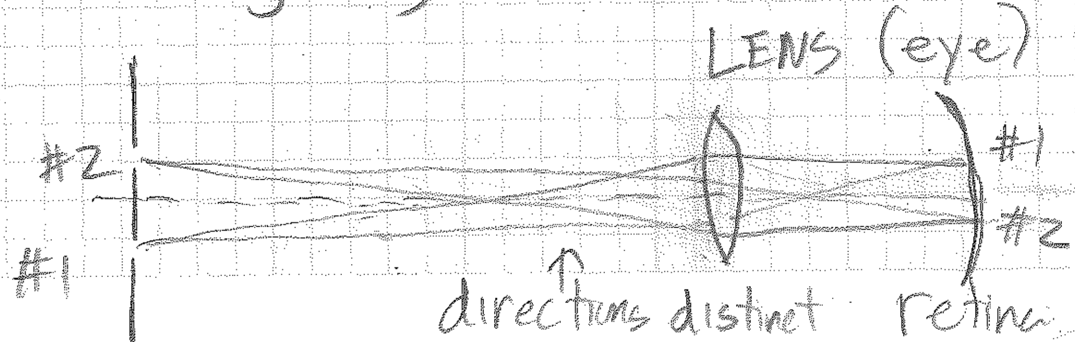
Example: $\Delta = 1$ mm
$a \simeq 0.1$ mm
$\lambda \simeq 550$ nm (center of visible).

can resolve lines (say, 1mm spacings on a meter stick) up to a distance of (by screen projection)

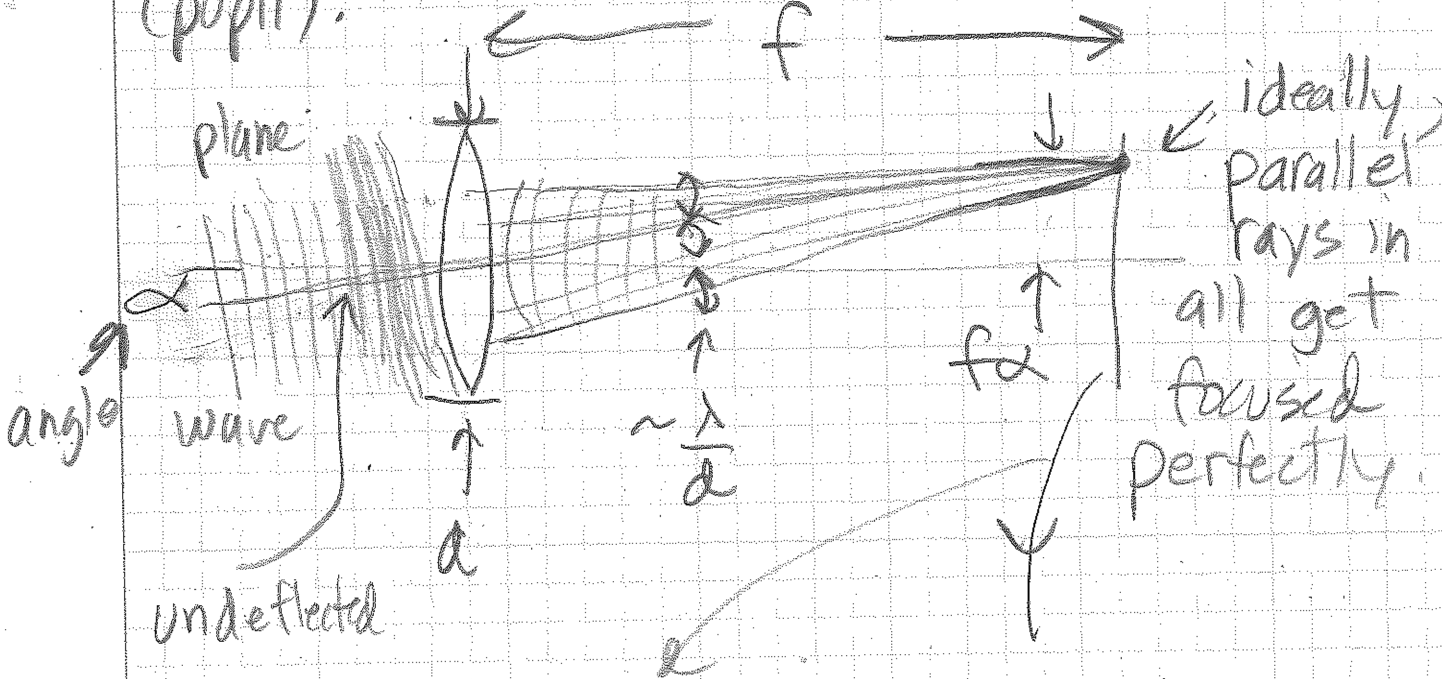
$$D \sim \frac{a}{\lambda}\Delta \sim \frac{10^{-4}}{550 \cdot 10^{-9}} \times 10^{-3} = \frac{1}{550 \cdot 10^{-2}}$$

$D \sim 18$ cm SMALL!

But ..... DIRECTION of beams still allows distinguishing of slits

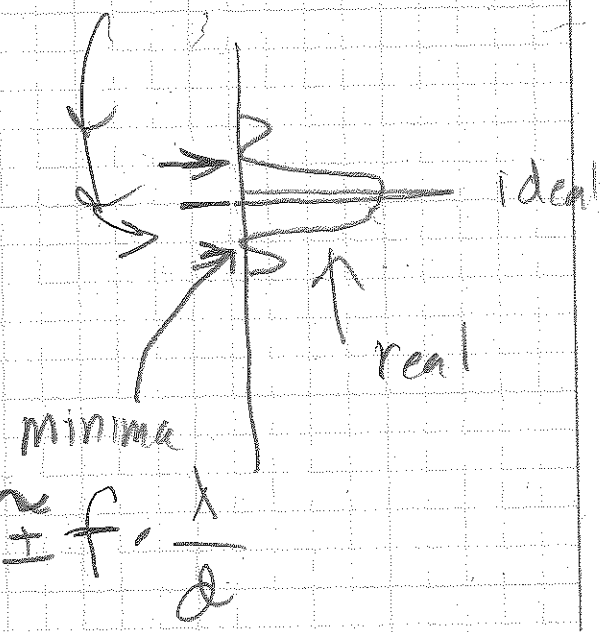


But the irreducible diffraction comes from the diameter of the eye's lens (pupil).

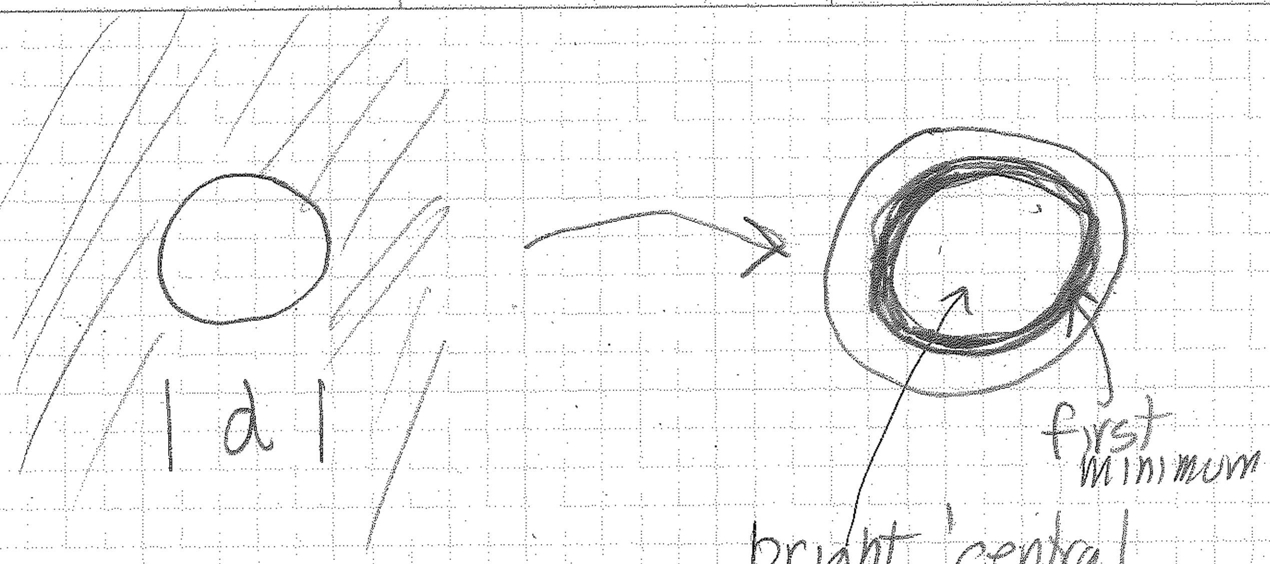


REALITY: angles messed up by $\sim \frac{\lambda}{d}$, d = aperture.

but actually, eyes don't have SLITS, but $\approx$ circlular apertures



Calculation of diffraction minimum a little harder for circle



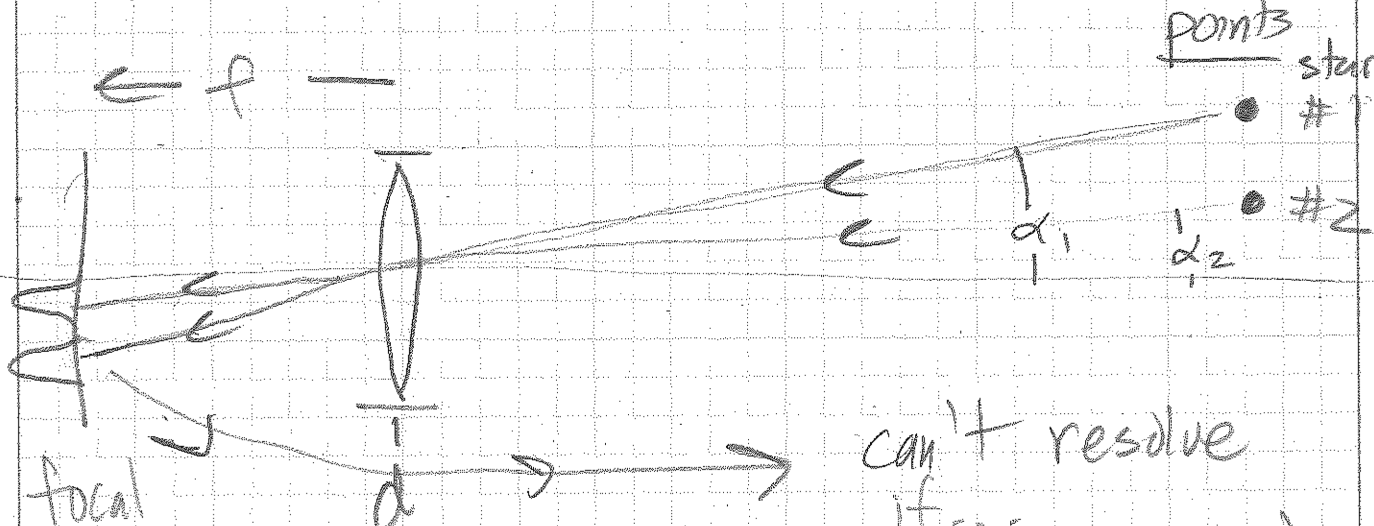
Circular Aperture

bright central "Airy" disk

$\theta \sim 1.22 \frac{\lambda}{d}$

the 1.22 comes from some integrals

How this works



eye or telescope

can't resolve if...

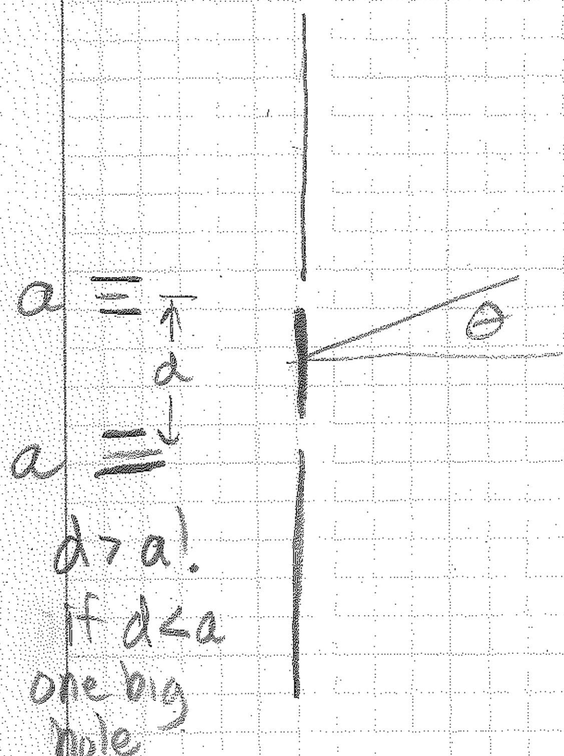
$|\alpha_1 - \alpha_2| \lesssim 1.22 \frac{\lambda}{d}$

want large d
(atmospherics actually limit $d \lesssim 1m$)

Diffraction + interference Together (coherent)

THEY MULTIPLY .. works since $a < d$

so $\frac{\lambda}{a} > \frac{\lambda}{d}$



$\sim \frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$ envelope

$\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$

$$I = I_{max}\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)\frac{\sin^2\left(\frac{\pi a}{\lambda}\sin\theta\right)}{\left(\frac{\pi a}{\lambda}\sin\theta\right)^2}$$

since $d < a$,
this "wiggles" inside

See Figures 16–18 of Text, pp. 977–979
(don't do derivation)

But, # fringes $\simeq 2\,\frac{\lambda/a}{\lambda/d} \simeq 2\,d/a$

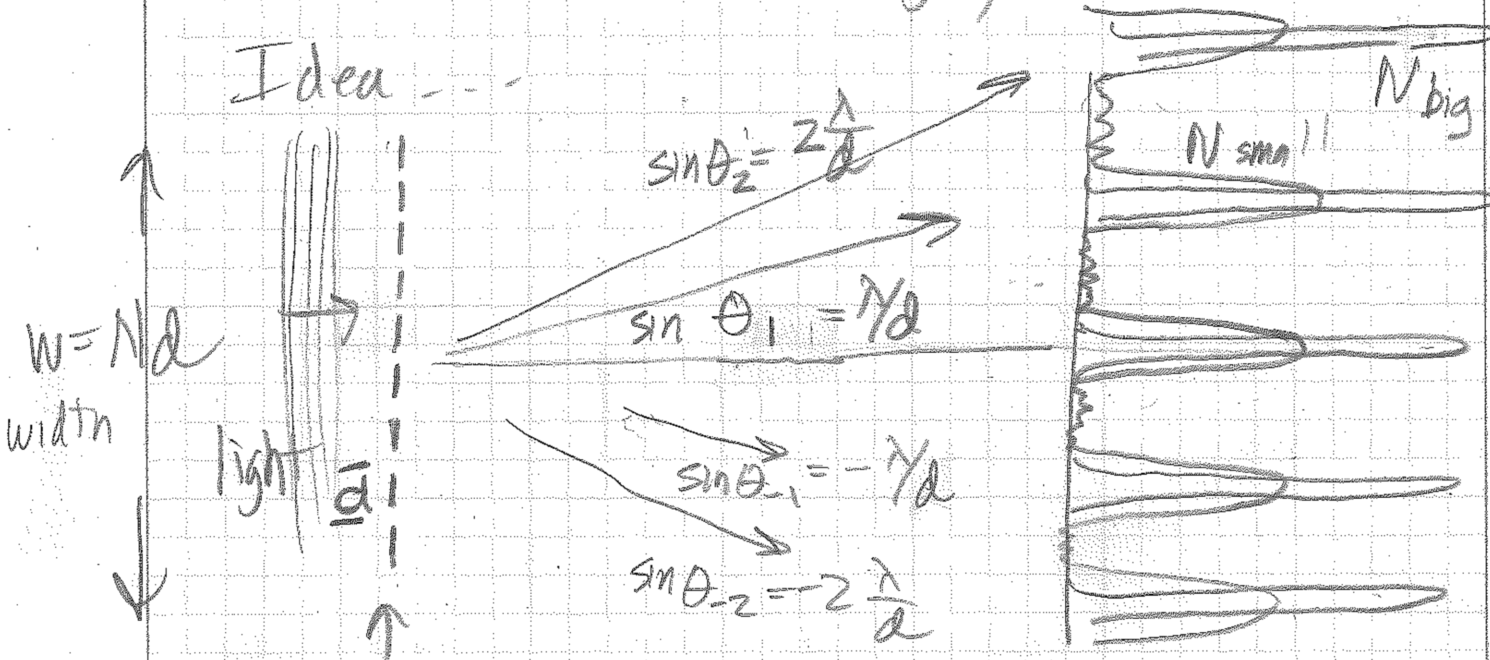
(could do demo .. 84.06)

Diffraction Grating

You have glasses with ~ 500 lines/mm → $d \sim 2\mu m$

about 10 mm high, ~ 5000 lines

Idea ---



many are better than 2, but, why? → • peaks get narrower

• $\theta_{max} \sim m\frac{\lambda}{d}$ ← called "order"

• actually capable of distinguishing different

full width, m^{th} order

• $\delta\theta_m = \frac{\lambda}{Nd\cos\theta_m}$

Narrowness of central peak (no phase difference)

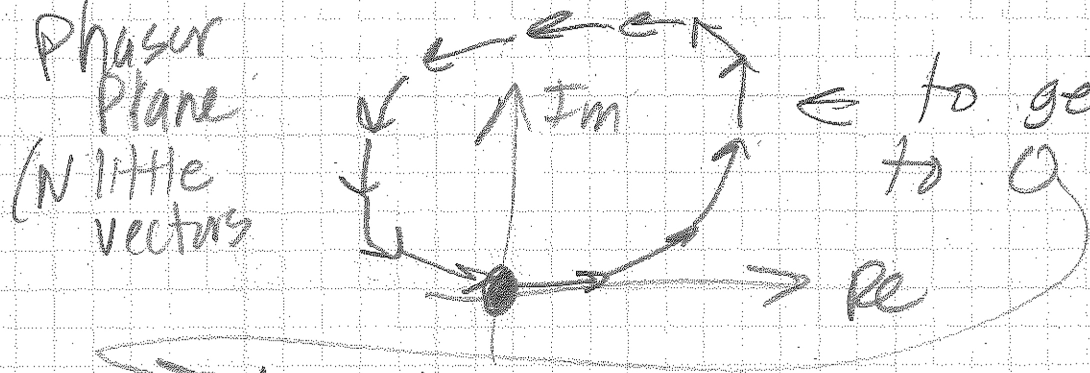
Max → → → → ... → →

slit 1 slit 2 slit 3 4 ... N-1 N

Min : each "phasor" in the complex plane gets $e^{ik\Delta x}$ now $\Delta x = d \sin(\delta\theta_0)$

$\delta\theta_0$ off 0

Minimum the all wrap around to 0



$\Rightarrow$ the tolerance on direction of each must get better as N increases to successfully end up back at 0, which is the minimum.

$$e^{iNk\Delta x} = e^{i2\pi}$$

$$\text{or } N \frac{2\pi}{\lambda} d \sin \delta\theta_0 = 2\pi$$

$$\sin \delta\theta_0 = \frac{\lambda}{Nd} = \frac{\lambda}{W}$$

very small

big

$$\delta\theta_0 \sim \frac{\lambda}{Nd} \sim \frac{\lambda}{W}$$

W = width of grating

Higher Order

$$\frac{2\pi}{\lambda} d \sin(\theta_m + \delta\theta_m) = \underset{\uparrow \text{ at max}}{2\pi m} + \underset{\uparrow \text{ increment to achieve zero}}{\frac{2\pi}{N}}$$

$$\sin\theta_m + \delta\theta_m \cos\theta_m$$

$$\cancel{\frac{2\pi}{\lambda} d \sin\theta_m} + \delta\theta_m \frac{2\pi}{\lambda} d\cos\theta_m = \cancel{2\pi m} + \frac{2\pi}{N}$$

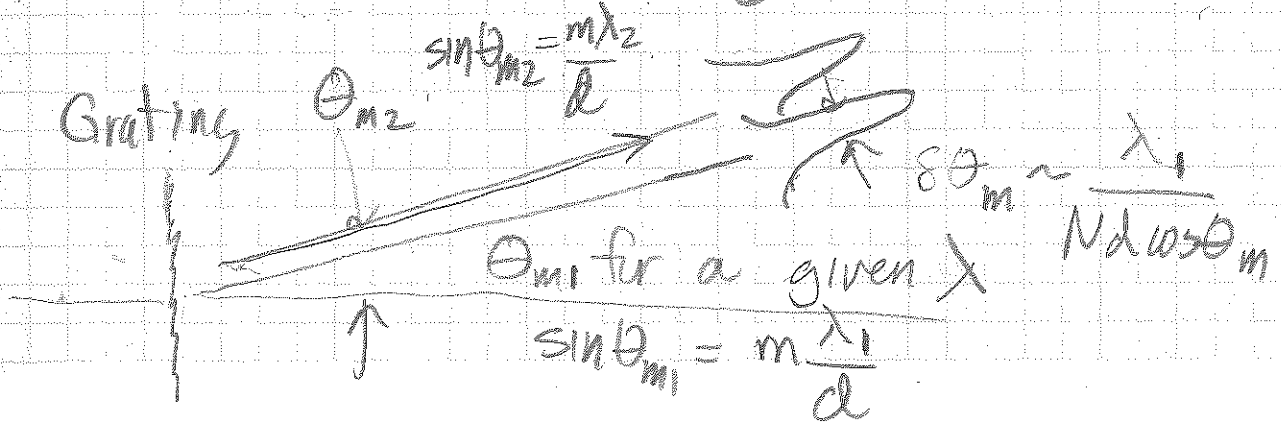
condition to be max

$$\boxed{\delta\theta_m = \frac{\lambda}{N d \cos\theta_m}}$$

because of $\cos\theta_m$, a little bigger than $\delta\theta_0$.

the real test of diffraction grating - -
what wavelengths can be resolved?

Dispersion + Resolving Power



Two distinguish the two peaks,
by how much larger must λ_2 be that λ_1?
$\delta\lambda = \lambda_2 - \lambda_1$

need to know.. $\frac{d\theta_m}{d\lambda}$

$$\sin\theta_m = m\frac{\lambda}{d}$$

$$\cos\theta_m \, d\theta_m = \frac{m}{d}\, d\lambda$$

this m really matters

$$\frac{d\theta_m}{d\lambda} = \frac{m}{d\cos\theta_m}$$

higher order, more sensitivity to changes in λ.

$$\delta\lambda = \frac{\delta\theta_m}{\left(\frac{d\theta_m}{d\lambda}\right)} = \frac{\frac{\lambda}{Nd\cos\theta_m}}{\frac{m}{d\cos\theta_m}}$$

$$\delta\lambda = \frac{\lambda}{mNd} = \frac{\lambda}{mW}$$

$\Rightarrow$ wider grating, smaller $\delta\lambda$ ☺
$\Rightarrow$ higher order, smaller $\delta\lambda$ ☺
$\rightarrow$ d? $\Rightarrow$ determines angle

Figure of merit... $\frac{\lambda}{\delta\lambda} = mW = mNd = R$

"Bigger Better" $\Rightarrow$ Resolving Power