Nelson 5.1

Thursday, May 01, 2008
5:27 PM

$$n(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\omega_p = 2 \times 10^{15}\, s^{-1}$$

a) $$K_p = \frac{\omega_p}{V_p} \stackrel{?}{=} \frac{\omega_p}{c}$$

$$K_p = \frac{2 \times 10^{15}\, s^{-1}}{3 \times 10^{10}\, cm/s} = \boxed{\frac{2}{3} \times 10^5 \frac{1}{cm}}$$

$$\lambda_p = \frac{2\pi}{k_p} = 9.4 \times 10^{-5}\, cm = \boxed{940\, nm}$$

b) $$\frac{V_p(\omega)}{c} = \frac{1}{1 - \omega_p^2/\omega^2} = \frac{1}{1 - \omega_p^2\left(\lambda^2/(2\pi)^2 c^2\right)}$$

$$\frac{\omega}{c} = \frac{2\pi}{\lambda}$$

$$\boxed{\frac{V_p(\omega)}{c} = \frac{1}{1 - \frac{\omega_p^2}{c^2}\frac{\lambda^2}{(2\pi)^2}}}$$

$$= \frac{1}{1 - k_p^2 \lambda^2/4\pi^2}$$

c) $v_{\ldots} = \frac{d\omega}{\ldots}$ [illegible]

c) $V_g = \frac{d\omega}{dK}$ $\qquad \omega\, n(\omega) = cK$

$$\omega - \frac{\omega_p^2}{\omega} = c\,K$$

$$d\omega - \frac{\omega_p^2(-1)}{\omega^2}\,d\omega = c\,dk$$

$$\frac{dK}{d\omega} = \frac{1}{C}\left(1 + \omega_p^2/\omega^2\right)$$

$$V_g = \frac{C}{1 + \omega_p^2/\omega^2}$$

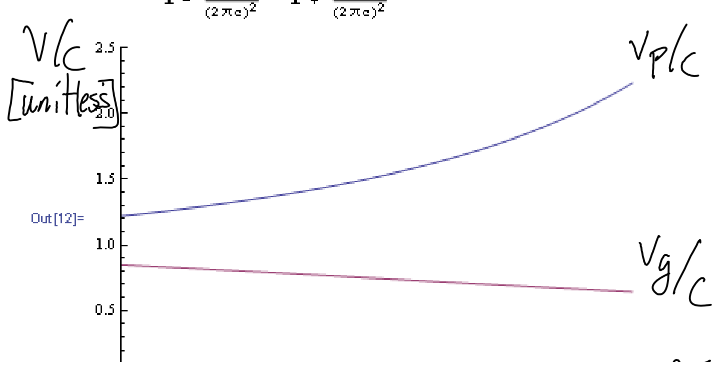
$$= \frac{C}{1 + \omega_p^2/\left(\frac{2\pi C}{\lambda}\right)^2}$$

$$\boxed{\frac{V_g}{C} = \frac{1}{1 + \frac{\omega_p^2 \lambda^2}{4\pi^2 C^2}}}$$

Untitled-1 *

In[10]:= ωp = 2 10^15;
c = 3 10^17; ← in nm/s
Plot[{ $\frac{1}{1 - \frac{\omega p^2 \lambda^2}{(2\pi c)^2}}$, $\frac{1}{1 + \frac{\omega p^2 \lambda^2}{(2\pi c)^2}}$ }, {λ, 400, 700}, PlotRange → {0, 2.5}]

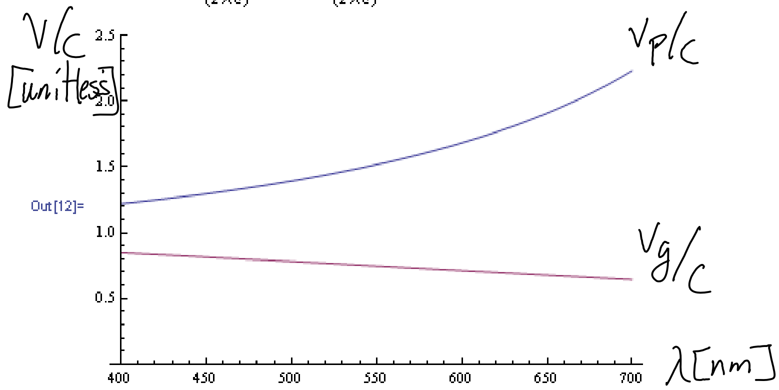
Out[12]=



Untitled-1 *

In[10]:= $\omega p = 2\, 10^{15}$;
$c = 3\, 10^{17}$; ← in nm/s

$$\text{Plot}\left[\left\{\frac{1}{1-\frac{\omega p^2\,\lambda^2}{(2\pi c)^2}},\ \frac{1}{1+\frac{\omega p^2\,\lambda^2}{(2\pi c)^2}}\right\},\ \{\lambda, 400, 700\},\ \text{PlotRange} \to \{0, 2.5\}\right]$$



phase velocity → ∞ as $\lambda \to \lambda_p$

group velocity → $\frac{1}{2}c$ as $\lambda \to \lambda_p$

" " → 0 as $\lambda \to \infty$

100%

Crawford 9.1

Thursday, May 01, 2008
6:23 PM

a) $L_0 \lambda \approx d^2$

$$L_0 \approx \frac{d^2}{\lambda} = \frac{(0.1 \times \cancel{10^{-3}}\, m)^2}{500 \times \cancel{10^{-9}}\, m}$$

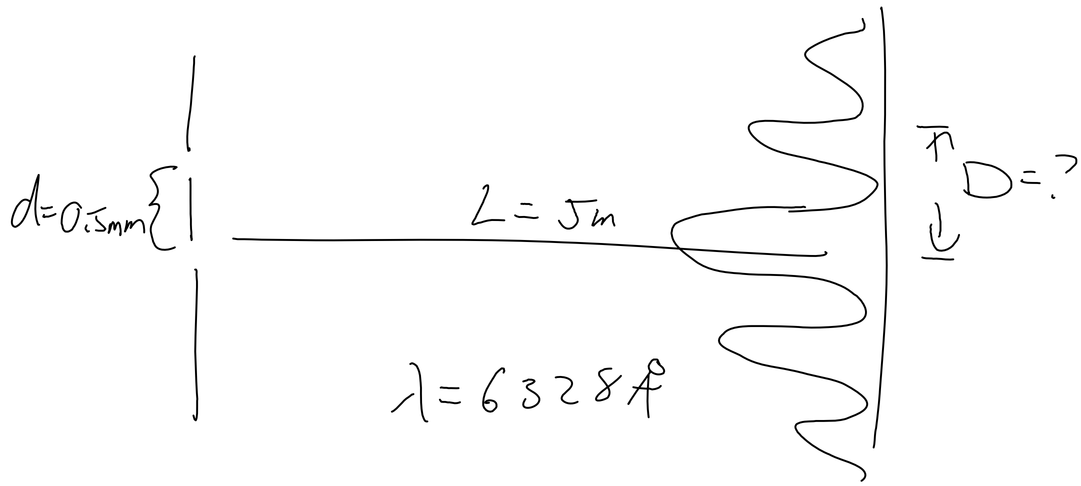
take $\lambda = 500\ nm$

$$= \frac{0.01\ m^{\cancel{2}}}{0.5\ \cancel{m}} = \boxed{.002\, m} \quad \boxed{0.2\, cm}$$

b) $L_0 = \frac{(10 cm)^2}{3 cm} = \boxed{33 cm}$

Crawford 9.2

Monday, May 12, 2008
12:55 PM



$$D \approx L\theta \simeq L\frac{\lambda}{d}$$

$$= \frac{\cancel{5}\,m \; 6328 \times 10^{\cancel{-10}\; \cancel{-7}\; -6}\; \cancel{m}}{\cancel{0.5} \times \cancel{10^{-3}}\; \cancel{m}}$$

$$= 6.328 \times 10^{-3}\, m$$

$$= \underline{6.328\ mm.}$$

Crawford 9.4

Monday, May 12, 2008
1:09 PM

Coherency condition was disc. in § 9.4 of Crawford

Basic condition is,

$$dD << L\lambda$$

$$L >> \frac{dD}{\lambda} = 0.9/m$$

Crawford 9.6

Monday, May 12, 2008
1:39 PM

Unaided eye means we use the diameter of our pupil for maximum angular resolution.

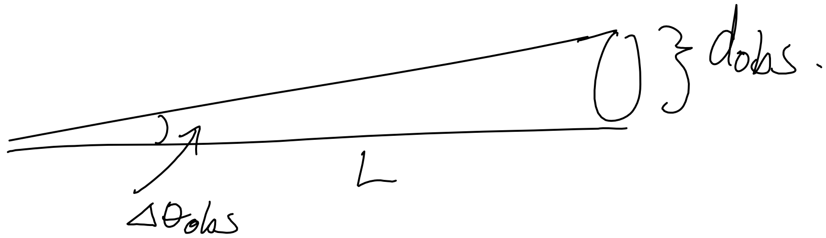
$$\Delta\theta_{min} = \frac{\lambda_{min}}{d_{max}}$$ ← we want find smallest angle we can resolve

$\lambda_{min} \approx 400\,nm$

$d_{max} \approx 0.8\,cm$ found on wikipedia

$$\Delta\theta_{min} = \boxed{5 \times 10^{-5}}$$

The $\Delta\theta_{observed} = \frac{d_{obs}}{L}$



$d_{obs} = \Delta\theta_{obs} \cdot L = \Delta\theta_{max} \cdot L$

this will make the object look like a point source.

$d_{obs} \approx \underline{4700\,mi}$

the unaided eye can actually see
the size of Venus. Of course white
light has a range of wavelengths
but use $\lambda_{min} = 700 nm$.
$d_{obs} \approx 8100$ mi. which is still ok,