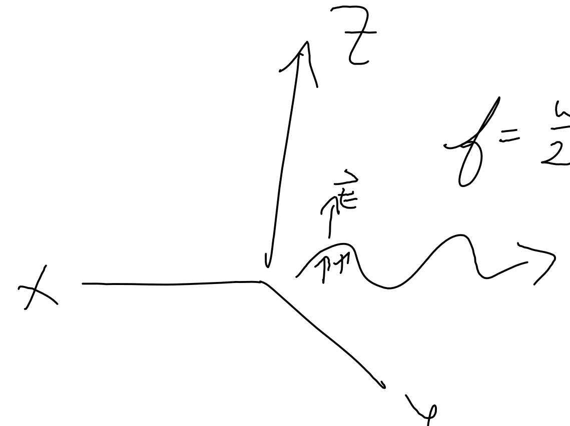
Purcell 9.7

Wednesday, April 23, 2008
11:18 AM



$f = \frac{\omega}{2\pi} = 10^8\ Hz$

$$\vec{E} = -2\hat{z}E_0 \sin\left(\frac{2\pi f x}{c}\right)\cos(2\pi f t)$$

$$\vec{B} = 2\hat{y}\,E_0 \cos\left(\frac{2\pi f x}{c}\right)\sin(2\pi f t)$$

Purcell 9.9

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9.9)

a) Energy density $= 4\times10^{-13}\,\text{erg/cm}^3 = \dfrac{\overline{E}^2}{4\pi}$

$$\Rightarrow \overline{E} = \sqrt{4\pi(4\times10^{-13})\tfrac{\text{erg}}{\text{cm}^3}} = 2.24\times10^{-6}\,\frac{\text{SV}}{\text{cm}}$$

$$\frac{2.24\times10^{-6}\,\text{SV}}{1\,\text{cm}}\cdot\frac{300\,\text{V}}{1\,\text{SV}}\cdot\frac{100\,\text{cm}}{1\,\text{m}} = 0.067\,\text{V/m}$$

b) energy density $= 4\times10^{-13}\,\text{erg/cm}^3$

power density = energy density (wave speed)

$$= \left(4\times10^{-13}\,\frac{\text{erg}}{\text{cm}^3}\right)3\times10^{10}\,\frac{\text{cm}}{\text{s}}$$

$$= \left(.012\,\frac{\text{erg}}{\text{cm}^2\text{sec}}\right)\left(\frac{1\text{ watt}}{10^7\,\text{erg/sec}}\right)\left(\frac{10{,}000\,\text{cm}^2}{\text{m}^2}\right)$$

$$= 1.2\times10^{-5}\,\text{W/m}^2$$

transmitted power is spread over a sphere

$$S_{transmitter} = \frac{10^3\,(\text{W})}{4\pi r^2}$$

$$1.2\times10^{-5}\,\frac{\text{W}}{\text{m}^2} = \frac{10^3\,\text{W}}{4\pi r^2}$$

$$r^2 = \frac{10^3}{1.2\times10^{-5}(4\pi)}\,\text{m}^2 \Rightarrow r = \sqrt{\frac{10^3}{1.2\times10^{-5}(4\pi)}}\,\text{m} = 2575\,\text{m} \approx 26\,\text{km}$$

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Purcell 9.10

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9.10

$$\int_c B \cdot d\vec{s} = \frac{1}{c}\int_S \left(\frac{\partial E}{\partial t} + 4\pi J\right) \cdot da$$

The capacitor is ~~discharging~~ $\Rightarrow$ E field is changing with time.

$$\sigma = \frac{Q}{\pi b^2} \qquad \boxed{J = 0}$$

$$E = 4\pi\sigma = \frac{4Q}{b^2}$$

$$\Rightarrow \frac{\partial E}{\partial t} = \frac{4}{b^2}\frac{\partial Q}{\partial t} = \frac{4I}{b^2}$$

$$\int_c B \cdot d\vec{s} = \frac{1}{c}\int_S \frac{\partial E}{\partial t} \cdot da$$

$$2\pi r B = \frac{1}{c}\int_S \left(\frac{4I}{b^2}\right) da$$

$$2\pi r B = \frac{4I}{cb^2}\int_S da = \frac{4I}{cb^2}(\pi r^2)$$

$$B = \frac{1}{\cancel{2}\cancel{\pi}\cancel{r}} \frac{\overset{2}{\cancel{4}}I(\cancel{\pi} r^{\cancel{2}})}{cb^2} = \underline{\frac{2Ir}{cb^2}}$$

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Purcell 10.1

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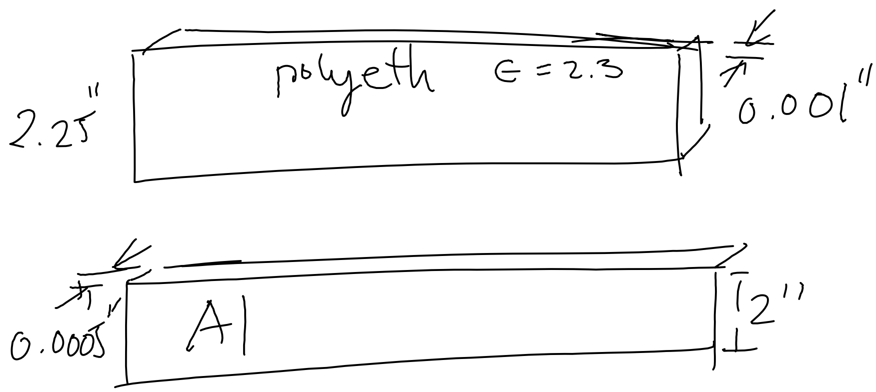
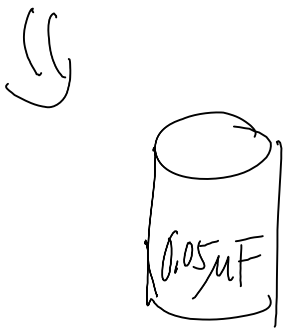
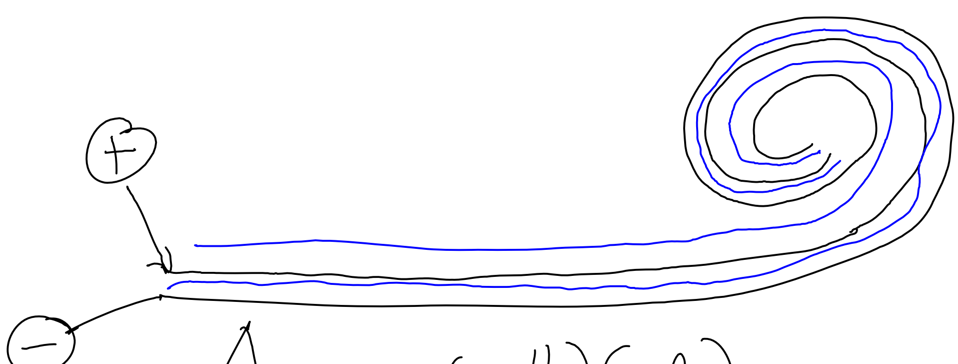




Stuck the dielectric & conductor together. + Lead on the beginning of conductor & − lead on the end. Start rolling. Top view.



$$C = \frac{A}{4\pi s} = \frac{(2'')(l)}{4\pi(0.001'')}\epsilon$$

$$= 2.3 \frac{\cancel{2} \times 10^{\cancel{3}} \, l}{2 \cancel{4} \pi} = 5 \times \cancel{10^{-8}} \cancel{F} \left(\frac{9 \times \cancel{10^{11}} \, cm}{1 \cancel{F}} \right)$$

$$\boxed{\begin{aligned} l &= \frac{45 \, cm \cdot 2\pi}{2.3} \\ &\approx 130 \, cm \\ \text{need} &\approx 1.3 \text{ m. of each type.} \end{aligned}}$$

Purcell 10.2

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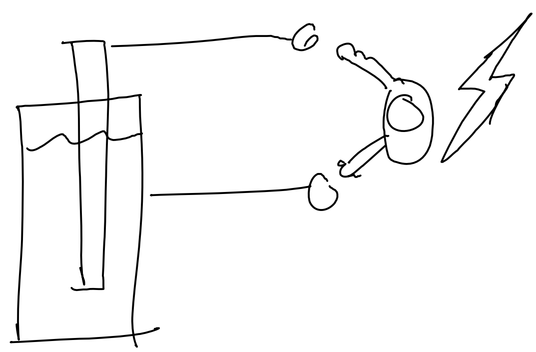


$$C = \frac{A}{4\pi s}\epsilon_w + \frac{A}{4\pi t}\epsilon_g$$

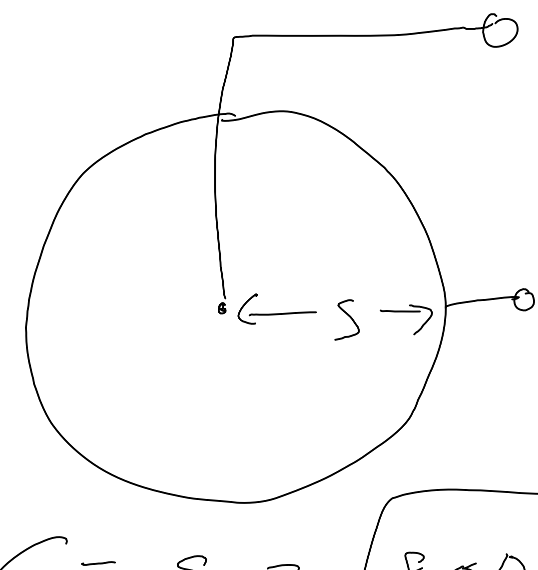
$A = 2\pi r h$, $v = 1000\,cm^3 = \pi r^2 h$

let $r = 5\,cm \rightarrow h = 12.7\,cm$

$A = 400\,cm^2$

$s = 5\,cm \rightarrow$ $C = \frac{100}{\pi}\left(\frac{88}{5} + \frac{4}{0.2}\right) cm$

$\boxed{\cong 800\,cm}$



$C = s = \boxed{800\,cm}$

Purcell 10.3

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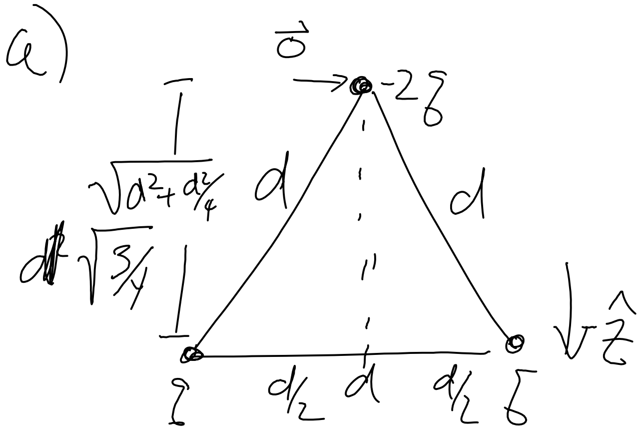
For point charge $\vec{p} = \sum_{i=1}^{n} q_i \vec{r}_i'$

a)

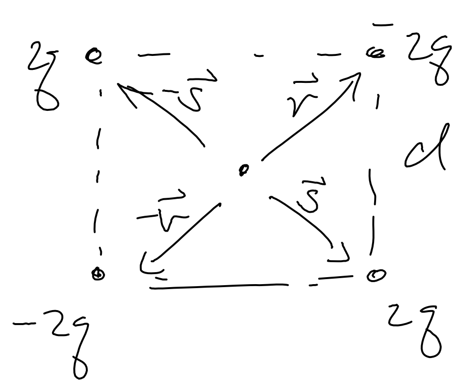


the left & right moment subtracts to zero

$$\vec{p} = -\cancel{2}q\, d\sqrt{3/4}\ \hat{z}$$

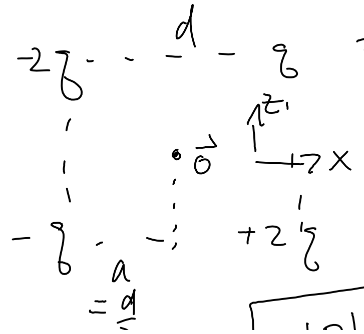
$$\boxed{= -q\, d\sqrt{5}\ \hat{z}}$$

b)



$$\boxed{\vec{p} = 0}$$

c)



$\vec{p} = aq(1+2+1+2)\hat{x}$

$aq(1+1-2-2)\hat{z}$

$\vec{p} = aq(6\hat{x} - 2\hat{z})$

$= d$

$-q \cdot -\hat{i} \quad +2q \quad p = aq(6x - 2z)$

$a = \frac{d}{2}$

$$\boxed{|P| = aq(\sqrt{40})}$$

Purcell 10.14

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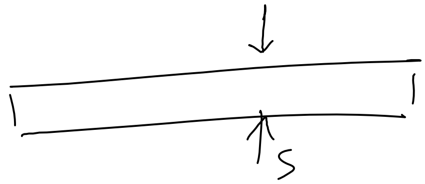
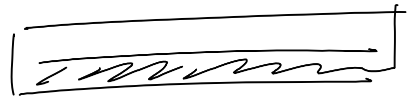
$$\varphi = E_{air} s$$



$$C_0 = \frac{A}{4\pi s} = \frac{Q}{\varphi}$$



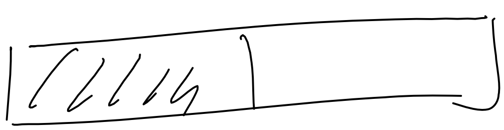
C_1

$$C_1 = \frac{Q}{\varphi_1}, \quad \varphi_1 = E_{Air} \cdot \frac{s}{2} + E_D \cdot \frac{s}{2}$$

$$E_D = E_{air} / \epsilon$$

$$C_1 = \frac{Q}{E_{air} s \left(\frac{1}{2} + \frac{1}{2\epsilon} \right)}$$

$$\boxed{= C_0 \frac{2}{\left(1 + \frac{1}{\epsilon}\right)}}$$



C_2

$$C_2 = \frac{A}{4\pi s} \frac{1}{2} + \frac{A}{4\pi s} \frac{1}{2} \epsilon$$

$$\boxed{= \frac{1}{2} C_0 (1 + \epsilon)}$$

Purcell 10.24

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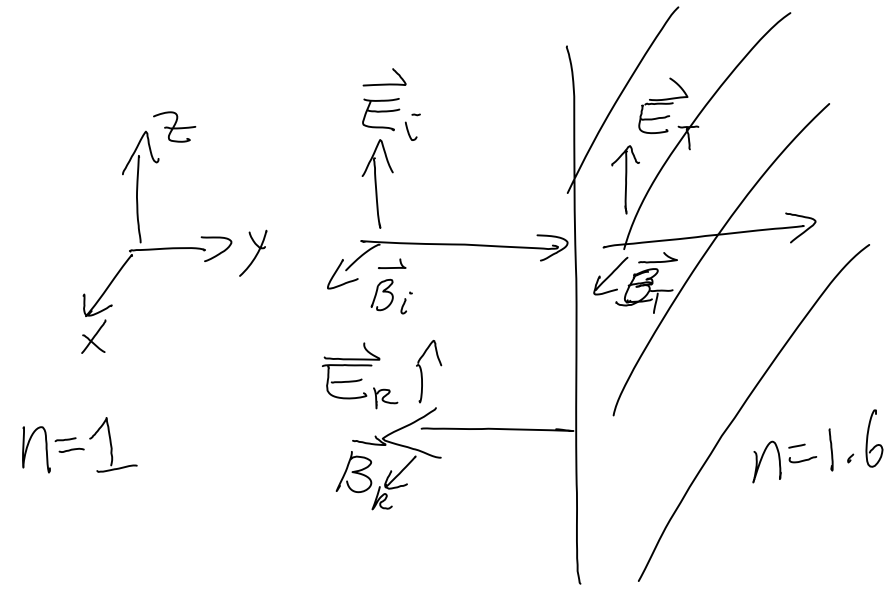

In
$$\vec{E}_i = E_i \sin(ky-\omega t)\,\hat{z}$$
$$\vec{B}_i = B_i \sin(ky-\omega t)\,\hat{x}$$

Ref
$$\vec{E}_r = E_r \sin(ky+\omega t)\,\hat{z}$$
$$\vec{B}_r = B_r \sin(ky+\omega t)\,\hat{x}$$

Trans.
$$\vec{E}_T = E_T \sin(k'y-\omega t)\,\hat{z}$$
$$\vec{B}_T = B_T \sin(k'y-\omega t)\,\hat{x}$$

This is all well and good but we're trying to find P_R/P_I! So we need $|E_R|/|E_I|$ somehow. Purcell tells you to apply the bdy cond. that $\vec{E}$ & $\vec{B}$ field must be continous at the interface. This must be true since $\vec{E}$ can only be discontinous if it pass through somewhere w/ electric charge ie

$$\vec{\nabla}\cdot\vec{E} = \rho_E \overset{\text{for this case.}}{\overset{\Downarrow}{=}} 0$$

Similiar argument follows for the $\vec{B}$-field

$$\vec{\nabla}\cdot\vec{B} = \rho_B \underset{\text{so far}}{=} 0$$

So

$$\left(\vec{E}_I = \vec{E}_R + \vec{E}_T\right)\Big|_{y=0}$$

eq (*) $$\left(\vec{B}_I = \vec{B}_R + \vec{B}_T\right)\Big|_{y=0}$$

Technically we have $6\times3=18$ unknowns and 6 equations. But $\vec{E}=E\hat{x}$ $\vec{B}=B\hat{z}$, so we only deal with 6 unknown w/ 2 equation. To get a ratio of $\frac{E_R}{E_I}$ we need 3 more equations. Magically we have can apply Maxwell's equation, $\vec{\nabla}\times\vec{E} = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}$, 3 times for each wave.

for example,

$$\vec{\nabla}\times\vec{E}_R = -\frac{1}{c}\frac{\partial}{\partial t}\vec{B}_R$$

R) $$kE_R\cos(ky+\omega t) = \frac{-1}{c}B_r\cos(ky+\omega t)\cdot\omega$$

$c=\frac{\omega}{k}$, & this equation must satified at all time. So we find,

$$\boxed{E_R = -B_R}$$

$$\boxed{E_R = B_R}$$

I) $$K E_I \cos(ky - \omega t) = -\frac{1}{c}(-\omega) B_I \cos(ky - \omega t)$$

$$\boxed{E_I = B_I}$$

T) $$k' E_T \cos(k'y - \omega t) = -\frac{1}{c}(-\omega) B_T \cos(k'y - \omega t)$$

$$\frac{k'}{k} = n$$

$$n E_T = B_T$$

plug in to eq (*), write B's in terms of E's

(1) $-E_I = E_R - E_T$

(2) $-E_I = -E_R - nE_T$

$$n(1) - (2) = (-n+1)E_I = (n+1)E_R$$

$$\frac{E_R}{E_I} = \frac{(n-1)}{(n+1)}$$

for $n = 1.6$

$$\boxed{\frac{P_R}{P_I} = \left|\frac{n-1}{n+1}\right|^2} \approx 0.05$$

This is the simplest version of Trans / Reflection problem. It will be repeated again & again in both future courses in E&M, QM, + anytime you have waves.