32. An atom in a state with zero angular momentum has spherical symmetry as far as its interaction with other atoms is concerned. It is sometimes called a “billiard-ball atom.” Explain.

33. “If the angular momentum of electrons in atoms were not quantized, the periodic table of the elements would not be what it is.” Discuss this statement.

34. How would the properties of helium differ if the electron had no spin, that is, if the only operative quantum numbers were \( n \), \( l \), and \( m_l \)?

35. We assert that the number of quantum numbers needed for a complete description of the motion of the electron in the hydrogen atom is equal to the number of degrees of freedom that the electron possesses. What is this number? How can you justify it?

36. Define and distinguish among the terms wave function, probability density, and radial probability density.

37. What are the dimensions and the SI units of a wave function, a probability density, and a radial probability density? Are the dimensions what you expect?

38. In the hydrogen atom state with \( l = 1 \), the spin and the orbital angular momentum vectors can be aligned either parallel or antiparallel. Which arrangement has the greater energy and why?

39. How can you account for the fact that in the state of the hydrogen atom with \( n = 2 \) and \( l = 0 \), the probability density is a maximum at \( r = 0 \) but the radial probability density is zero there? See Fig. 17.

40. Figure 18a shows the three probability densities for the hydrogen atom states with \( n = 2 \) and \( l = 1 \). What determines the direction in space that we choose for the \( z \) axis?

41. Consider the three probability density “dot plots” of Fig. 18a, each of which is a figure of revolution about the \( z \) axis. Do you see any connection between these figures and the semiclassical vector model of the atom (Fig. 9) for the case of \( l = 1 \)?

42. Use Heisenberg’s uncertainty principle to show that the probability densities in an \( l = 2 \) state have cylindrical symmetry about the \( z \) axis.

43. Explain how the interaction between the spin and the orbital motions of the valence electron in sodium leads to the splitting of the spectral lines of sodium, producing the familiar sodium doublet. See Fig. 19.

PROBLEMS

Section 51-1 The Bohr Theory

1. (a) By direct substitution of numerical values of the fundamental constants, verify that the energy of the ground state of the hydrogen atom is \(-13.6 \text{ eV}\) (see Eq. 18). Similarly, from Eq. 17 show that the value of the Rydberg constant \( R \) is \(0.01097 \text{ nm}^{-1}\). (c) Also verify the numerical value of \( a_0 \) by direct computation of its expression given in Eq. 19.

2. Answer the questions of Sample Problem 2, but for the Lyman series.

3. Using the Balmer–Rydberg formula, Eq. 3, calculate the five longest wavelengths of the Balmer series.

4. What are the \( (a) \) wavelength, \( (b) \) momentum, and \( (c) \) energy of the photon that is emitted when a hydrogen atom undergoes a transition from the state \( n = 3 \) to \( n = 1 \)?

5. Show, on an energy-level diagram for hydrogen, the quantum numbers corresponding to a transition in which the wavelength of the emitted photon is 121.6 nm.

6. (a) If the angular momentum of the Earth due to its motion around the Sun were quantized according to Bohr’s relation, what would be the nature and the relative magnitudes of the possible orbital angular momenta?

7. A hydrogen atom is excited from a state with \( n = 1 \) to one with \( n = 4 \). (a) Calculate the energy that must be absorbed by the atom. (b) Calculate and display an energy-level diagram showing the different photon energies that may be emitted if the atom returns to the \( n = 1 \) state.

8. The lifetime of an electron in the state \( n = 2 \) in hydrogen is about 10 ns. What is the uncertainty in the energy of the \( n = 2 \) state? Compare this with the energy of this state.

9. A diatomic gas molecule consists of two atoms of mass \( m \) separated by a fixed distance \( d \) rotating about an axis as indicated in Fig. 22. Assuming that its angular momentum is quantized as in the Bohr atom, determine \( (a) \) the possible angular velocities and \( (b) \) the possible rotational energies. (c) Calculate, according to this model, the ground-state energy, in eV, of an \( \text{O}_2 \) molecule for which \( d = 121 \text{ pm} \) and \( m = 16.0 \text{ u} \).

10. If an electron is revolving in an orbit at frequency \( v_0 \), classical electromagnetism predicts that it will radiate energy not
14. In Table 2 show that the quantity in the last column is given by

\[ \frac{100(v - v_0)}{v} = \frac{150}{n} \]

for large quantum numbers.

15. A neutron, with kinetic energy of 6.0 eV, collides with a resting hydrogen atom in its ground state. Show that this collision must be elastic (that is, energy must be conserved).

(Hint: Show that the atom cannot be raised to a higher excitation state as a result of the collision.)

16. (a) Calculate, according to the Bohr model, the speed of the electron in the ground state of the hydrogen atom. (b) Calculate the corresponding de Broglie wavelength. (c) Comparing the answers to (a) and (b), find a relation between the de Broglie wavelength \( \lambda \) and the radius \( a_0 \) of the ground-state Bohr orbit.

17. According to the correspondence principle, as \( n \to \infty \) we expect classical results in the Bohr atom. Hence the de Broglie wavelength associated with the electron (a quantum result) should get smaller compared with the radius of the Bohr orbit as \( n \) increases. Indeed, we expect that \( \lambda/r \to 0 \) as \( n \to \infty \). Show that this is the case.

18. A hydrogen atom in a state having a binding energy (the energy required to remove an electron) of 0.85 eV makes a transition to a state with an excitation energy (the difference in energy between the state and the ground state) of 10.2 eV. (a) Find the energy of the emitted photon. (b) Show this transition on an energy-level diagram for hydrogen, labeling with the appropriate quantum numbers.

19. From the energy-level diagram for hydrogen, explain the observation that the frequency of the second Lyman-series line is the sum of the frequencies of the first Lyman-series line and the first Balmer-series line. This is an example of the empirically discovered Ritz combination principle. Use the diagram to find some other valid combinations.

20. Calculate the recoil speed of a hydrogen atom, assumed initially at rest, if the electron makes a transition from the \( n = 4 \) state directly to the ground state. (Hint: Apply conservation of linear momentum.)

21. (a) How much energy is required to remove the electron from a He\(^+\) ion in its ground state? (b) From a Li\(^+\) ion in a state with \( n = 37 \)? (Hint: See Eq. 18.)

22. In stars the Pickering series is found in the He\(^+\) spectrum. It is believed to occur when a positively charged He\(^+\) ion loses an electron. Explain this statement. (Hint: See Eq. 18.)

23. In a hydrogen atom the electron is quantized as \( L = n \hbar \). Determine the angular momentum of the electron in its ground state.

24. In the ground state of the hydrogen atom, according to Bohr's theory, what are (a) the quantum number, (b) the orbital angular momentum, (c) the angular velocity, (d) the linear momentum, (e) the linear speed, (f) the force on the electron, (g) the acceleration of the electron, (h) the kinetic energy, (i) the potential energy, and (j) the total energy?

25. How do the quantities (b) to (k) in Problem 24 vary with the quantum number \( n \)?

26. Suppose that we wish to test the possibility that electrons in atoms move in orbits by "viewing" them with photons with sufficiently short wavelength, say 10.0 pm. (a) What would be the energy of such photons? (b) How much energy would such a photon transfer to a free electron in a head-on Compton collision? (c) What does this tell you about the possibility of confirming orbital motion by "viewing" an atomic electron at two or more points along its path? Assume that the speed of the electron is 0.10c.

27. Bohr proposed that, as an alternative to the correspondence principle, the quantization expression for the angular momentum (Eq. 20) could be taken as a basic postulate. Starting from this point, and using only classical results, derive Bohr's expression for the quantized energies of the stationary states of the hydrogen atom (Eq. 18).

28. (a) Calculate the wavelength intervals over which the Lyman, the Balmer, and the Paschen series extend. (The interval extends from the longest wavelength to the series limit.) (b) Find the corresponding frequency intervals.

Section 51.3 Angular Momentum

29. Verify that \( \mu_B = 9.274 \times 10^{-24} \) J/T = \( 5.788 \times 10^{-5} \) eV/T, as reported in Eq. 30.

30. If an electron in a hydrogen atom is in a state with \( l = 5 \), what is the smallest angle possible between \( \mathbf{L} \) and \( \mathbf{L}_z \)?

31. For a hydrogen atom in a state with \( l = 3 \), calculate the allowed values of (a) \( L_z \), (b) \( \mu_z \), and (c) \( \theta \). Find also the magnitudes of (d) \( \mathbf{L} \) and (e) \( \mu \). Where appropriate, express answers in units of \( \hbar \) and \( \mu_B \).

32. (a) Show that the magnetic moments of the electrons in the various Bohr orbits are given, according to the Bohr theory, by

\[ \mu = n \mu_B \]

in which \( \mu_B \) is the Bohr magneton and \( n = 1, 2, 3, \ldots \)

(b) How does this expression compare with the actual values?
(b) the orbital angular momenta are \( m \hbar \).

If the hydrogen atom, according to (a) the quantum number, (b) the angular momentum, (c) the linear momentum, (d) the linear momenta, and (e) the angular momenta, (f) the linear speed, (g) the acceleration of the electron, (h) the acceleration of the potential energy, and

(b) to (k) in Problem 24 vary with the angle, we can test the possibility that electrons in orbit around a nucleus are quantized by "viewing" them with photons with energies of 10.0 keV. (a) What would be the wavelength of these wavelengths? (b) How much energy would be needed to cause the transition of a free electron in a head-on collision with a proton? (c) What does this tell you about the possibility of accelerating electrons by "viewing" an atomic electron, if it happens to be in orbit around a nucleus? Assume that the speed of light is 3.00 \( \times \) 10^8 m/s.

An alternative to the correspondence between the quantization expression for the angular momentum of a free electron and the quantization of energy in the atom was proposed by de Broglie in 1924. The quantization of the energy levels of the electron in the atom is given by the equation:

\[ E_n = n^2 \hbar^2 k / 8 m \]

where \( E_n \) is the energy of the electron in the nth level, \( n \) is the quantum number, \( \hbar \) is Planck's constant, \( k \) is the Boltzmann constant, and \( m \) is the mass of the electron.

Calculate the energy of an electron in the nth level of a hydrogen atom.

The wavelength of the electron is given by the de Broglie equation:

\[ \lambda = \frac{h}{p} \]

where \( h \) is Planck's constant and \( p \) is the momentum of the electron.

Calculate the wavelength of the electron at the nth level of the hydrogen atom.

Section 51.4 - The Stern-Gerlach Experiment

35. Of the three scalar components of \( \mathbf{L} \), one, \( L_z \), is quantized, according to Eq. 25. In view of the restrictions imposed by Eqs. 23 and 24, taken together, show that the most that can be said about the other two components of \( \mathbf{L} \) is

\[ \sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m^2} \hbar \]

Note that these two components are not separately quantized. Show also that

\[ \sqrt{L_x^2 + L_y^2} = \sqrt{l(l+1) - m^2} \hbar \]

Correlate these results with Fig. 9.

36. Suppose a hydrogen atom (in its ground state) moves 82 cm in a direction perpendicular to a magnetic field that has a gradient, in the vertical direction, of 16 mT/m. (a) What is the force on the electron due to the magnetic moment of the electron, which we take to be 1 Bohr magneton? (b) Find its vertical displacement if its speed is 970 m/s.

37. Calculate the acceleration of the silver atom as it passes through the deflecting magnet in the Stern-Gerlach experiment of Sample Problem 6.

38. Assume that in the Stern-Gerlach experiment described for neutral silver atoms the magnetic field \( \mathbf{B} \) has a magnitude of 520 mT. (a) What is the energy difference between the orientations of the silver atoms in the two subbeams? (b) What is the frequency of the radiation that would induce a transition between these two states? (c) What is its wave length, and to what part of the electromagnetic spectrum does it belong? The magnetic moment of a neutral silver atom is 1 Bohr magneton.

Section 51.5 - The Ground State of Hydrogen

49. In the ground state of the hydrogen atom, evaluate the square of the wave function, \( \psi^2(r) \), and the radial probability density \( P_r(r) \) for the positions \( (a) r = 0 \) and \( (b) r = a_0 \). Explain what these quantities mean.

50. In Fig. 16b, verify the plotted values of \( P_r(r) \) at \( r = 0 \), \( r = a_0 \), and \( r = 2a_0 \).

51. Find the ratio of the probabilities of finding the electron in the hydrogen atom in a thin shell at the Bohr radius to that of finding it in a shell of the same thickness at twice that distance.

52. A spherical region of radius 0.05 \( a_0 \) is located a distance \( a_0 \) from the nucleus of a hydrogen atom in its ground state. Calculate the probability that the electron will be found inside this sphere. (Assume that \( \psi \) is constant inside the sphere.)

53. For a hydrogen atom in its ground state, calculate the probability of finding the electron between two spheres of radii \( r = 1.00a_0 \) and \( r = 0.80a_0 \).

54. In atoms there is a finite, though very small, probability that, at some instant, an orbital electron will actually be found inside the nucleus. In fact, some unstable nuclei use this occasional appearance of the electron to decay by electron capture. Assuming that the proton itself is a sphere of radius 1.1 \( \times \) 10^-15 m and that the hydrogen atom electron wave function is the same as that of the free electron in the hydrogen atom, calculate the probability of finding the electron inside the proton.
\[ P = 1 - e^{-2x}(1 + 2x + 2x^2) \]

in which \( x = r/a_0 \). (b) Evaluate the probability that, in the ground state, the electron lies within a sphere of radius \( a_0 \).

57. Use the result of Problem 56 to calculate the probability that the electron in a hydrogen atom, in the ground state, will be found between the spheres \( r = a_0 \) and \( r = 2a_0 \).

58. For an electron in the ground state of the hydrogen atom, calculate the radius of a sphere for which the probability that the electron will be found inside the sphere equals the probability that the electron will be found outside the sphere. (Hint: See Problem 56.)

**Section 51-8 The Excited States of Hydrogen**

59. For the state \( n = 2, l = 0 \), (a) locate the two maxima for the radial probability density curve of Fig. 17b, and (b) calculate the values of the radial probability density at the two maxima; compare with Fig. 17b.

60. Using Eq. 46, show that, for the hydrogen atom state with \( n = 2 \) and \( l = 1 \),

\[ \int_0^\infty P_r(r) \, dr = 1. \]

What is the physical interpretation of this result?

61. For a hydrogen atom in a state with \( n = 2 \) and \( l = 0 \), calculate the probability of finding the electron between two spheres of radii \( r = 5.00a_0 \) and \( r = 5.01a_0 \).

62. For a hydrogen atom in a state with \( n = 2 \) and \( l = 0 \), what is the probability of finding the electron somewhere within the smaller of the two maxima of its radial probability density function? See Fig. 17b.

**Section 51-9 Details of Atomic Structure**

63. Potassium (\( Z = 19 \)), like sodium (\( Z = 11 \)), is an alkali metal, its single valence electron moving around a filled 18-electron argon-like core. As in sodium, there is a potassium doublet, its wavelengths being 764.5 nm and 769.9 nm. The quantum numbers of the levels that give rise to these lines are just the same as for sodium (see Fig. 19) except that \( n = 4 \). Calculate (a) the energy splitting between the two upper states and (b) the energy difference between the uppermost state and the ground state.

64. The wavelengths of the lines of the sodium doublet (see Fig. 19) are 588.995 nm and 589.592 nm. (a) What is the difference in energy between the two upper levels in that figure? (b) This energy difference comes about because the electron’s spin magnetic dipole moment can be oriented either parallel or antiparallel to the internal magnetic field associated with the electron’s orbital motion. Use the result you have just calculated to find the strength of this internal magnetic field. The electron’s spin magnetic dipole moment has a magnitude of 1 Bohr magneton.

65. Apply Bohr’s model to a muonic atom, which consists of a nucleus of charge \( Ze \) with a negative muon (an elementary particle with a charge \( q = -e \) and a mass \( m = 207m_e \), where \( m_e \) is the electron mass) circulating about it. Calculate (a) the muon–nucleus separation in the first Bohr orbit, (b) the ionization energy, and (c) the wavelength of the most energetic photon that can be emitted. Assume that the muon is circulating about a hydrogen nucleus (\( Z = 1 \)). See “The Muonium Atom,” by Vernon W. Hughes, Scientific American, April 1966, p. 93.

66. Apply Bohr’s model to the positronium atom. This consists of a positive and a negative electron revolving around their center of mass, which lies halfway between them. (a) What relationship exists between this spectrum and the hydrogen spectrum? (b) What is the radius of the ground-state orbit? (Hint: Calculate the reduced mass of the atom.) See “Exotic Atoms,” by E. H. S. Burhop, Contemporary Physics, July 1970, p. 335.