33. Distinguish between the Planck relation $E = nh\nu$ (Eq. 7) and the Einstein relation $E = hv$ (Eq. 17).

34. A photon has no rest mass since it can never be at rest with respect to any observer. If energy equals $mc^2$, how can a photon have any energy?

35. The momentum $p$ of a photon is given by $p = h/\lambda$. Why is it that, the speed of light, does not appear in this expression?

36. In discussing the propagation of light we sometimes use straight rays, sometimes waves, and still other times discrete photons. To what extent, if at all, are these views compatible with one another? Are there cases in which one view is clearly superior to the others?

37. Given that $E = hv$ for a photon, the Doppler shift in frequency of radiation from a receding light source would seem to indicate a reduced energy for the emitted photons. Is this in fact true? If so, what happened to the conservation of energy principle? (See "Questions Students Ask," *The Physics Teacher*, December 1983, p. 616.)

38. Photon $A$ has twice the energy of photon $B$. What is the ratio of the momentum of $A$ to that of $B$?

39. How does a photon differ from a material particle?

40. What is the direction of a Compton scattered electron with maximum kinetic energy compared with the direction of the incident monochromatic photon beam?

41. Why, in the Compton scattering picture (Fig. 14), would you expect $\Delta \lambda$ to be independent of the materials of which the scatterer is composed?

42. Why don’t we observe a Compton effect with visible light?

43. Light from distant stars is Compton scattered many times by free electrons in outer space before reaching us. This shifts the light toward the red. How can this shift be distinguished from the Doppler red shift due to the motion of receding stars?

44. In both the photoelectric effect and the Compton effect there is an incident photon and an ejected electron. What is the difference between these two effects?

45. List and discuss the assumptions made by Planck in connection with the cavity radiation problem, by Einstein in connection with the photoelectric effect, and by Compton in connection with the Compton effect.

46. Describe several experimental methods that can be used to determine the value of the Planck constant $h$.

**PROBLEMS**

**Section 49-1 Thermal Radiation**

1. In 1983 the Infrared Astronomical Satellite (IRAS) detected a cloud of solid particles surrounding the star Vega, radiating maximally at a wavelength of 32 $\mu$m. What is the temperature of this cloud of particles? Assume an emissivity of unity.

2. Low-temperature physicists would not consider a temperature of 2.0 mK (0.0020 K) to be particularly low. At what wavelength is the spectral radiacy of a cavity at this temperature a maximum? To what region of the electromagnetic spectrum does this radiation belong? What are some of the practical difficulties of operating a cavity radiator at such a low temperature?

3. Calculate the wavelength of maximum spectral radiancy and identify the region of the electromagnetic spectrum to which it belongs for each of the following: (a) The 2.7-K cosmic background radiation, a remnant of the primordial fireball. (b) Your body, assuming a skin temperature of 34°C. (c) A tungsten lamp filament at 1800 K. (d) The Sun, at an assumed surface temperature of 5800 K. (e) An exploding thermonuclear device, at an assumed fireball temperature of 10$^9$ K. (f) The universe immediately after the Big Bang, at an assumed temperature of 10$^8$ K. Assume cavity radiation conditions throughout.

4. (a) The effective surface temperature of the Sun is 5800 K. At what wavelength would you expect the Sun to radiate most strongly? In what region of the spectrum is this? Why then does the Sun appear yellow? (b) At what temperature is cavity radiation most visible to the human eye? See Fig. 1 in Chapter 42.

5. A cavity whose walls are held at 1900 K has a small hole, 1.00 mm in diameter, drilled in its wall. At what rate does energy escape through this hole from the cavity interior?

6. Calculate the thermal power radiated from a fireplace assuming an emissivity of 0.90, an effective radiating surface of 0.50 m$^2$, and a radiating temperature of 500°C. Does your answer seem reasonable?

7. (a) Show that a human body of area 1.80 m$^2$, emissivity $\varepsilon = 1.0$, and temperature 34°C emits radiation at the rate of 910 W. (b) Why, then, do people not glow in the dark?

8. A cavity at absolute temperature $T_1$ radiates energy at a power level of 12.0 mW. At what power level does the same cavity radiate at temperature $2T_1$?

9. A cavity radiator has its maximum spectral radiancy at a wavelength of 25.0 $\mu$m, in the infrared region of the spectrum. The temperature of the body is now increased so that the radiant intensity $I(T)$ of the body is doubled. (a) What is this new temperature? (b) At what wavelength will the spectral radiancy now have its maximum value?

10. A 100-W incandescent lamp has a coiled tungsten filament whose diameter is 0.42 mm and whose extended length is 33 cm. The effective emissivity under operating conditions is 0.22. Find the operating temperature of the filament.

11. An oven with an inside temperature $T_o = 215°C$ is in a room with a temperature of $T_r = 26.2°C$. There is a small opening of area $A = 5.20 \text{ cm}^2$ in one side of the oven. How much net power is transferred from the oven to the room? (Hint: Consider both oven and room as cavities with $\varepsilon = 1.0$.)

12. A thermograph is a medical instrument used to measure radiation from the skin. For example, normal skin radiates at a temperature of about 34°C and the skin over a tumor radiates at a slightly higher temperature. (a) Derive an approximate expression for the fractional difference $\Delta I/I$ in the radiant intensity between adjacent areas of the skin that are at slightly different temperatures $T$ and $T + \Delta T$. (b) Evaluate this expression for a temperature difference...
of 1.3 C°. Assume that the skin radiates with a constant emissivity.

13. A convex lens 3.8 cm in diameter and of focal length 26 cm produces an image of the Sun on a thin black screen the same size as the image. Find the highest temperature to which the screen can be raised. The effective temperature of the Sun is 5800 K.

14. The filament of a particular 100-W light bulb is a cylindrical wire of tungsten 0.280 mm in diameter and 1.80 cm long. See Appendix D for needed data on tungsten. Assume an emissivity of unity and ignore absorption of energy by the filament from the surroundings. (a) Calculate the operating temperature of the filament. (b) How long does it take for the filament to cool by 500 C° after the bulb is switched off?

15. Consider a planet, with radius R, revolving about the Sun in a circular orbit of radius r. Suppose that the planet has no atmosphere (and therefore no "greenhouse effect" on its surface temperature). (a) Show that the surface temperature T of the planet is given from the relation \( T^4 = \frac{P_{\text{sun}}}{16\pi ar^2} \), where \( P_{\text{sun}} \) is the radiant power output of the Sun. (b) Evaluate the temperature numerically for the Earth.

Section 49-2 Planck's Radiation Law

16. Show that the wavelength \( \lambda_{\text{max}} \) at which Planck's spectral radiation law, Eq. 6, has its maximum is given by Eq. 4:

\[
\lambda_{\text{max}} = \frac{2898 \text{ } \mu \text{m} \cdot \text{K}}{T}.
\]

(Hint: Set \( \frac{dI}{dT} = 0 \); an equation will be encountered whose numerical solution is 4.965.)

17. (a) By integrating the Planck radiation law, Eq. 6, over all wavelengths, show that the power radiated per square meter of a cavity surface is given by

\[
I(T) = \left( \frac{2\pi^2 k^4}{15h^2} \right) T^4 = \sigma T^4.
\]

(Hint: Make a change in variables, letting \( x = hc/\lambda kT \). The definite integral

\[
\int_0^\infty \frac{e^x}{e^x + 1} \, dx
\]

will be encountered, which has the value \( \pi^4/15 \).) (b) Verify that the numerical value of the constant \( \sigma \) is \( 5.67 \times 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4) \).

18. (a) An ideal radiator has a spectral radiance at 400 nm that is 3.50 times its spectral radiance at 200 nm. What is its temperature? (b) What would be its temperature if its spectral radiance at 200 nm were 3.50 times its spectral radiance at 400 nm?

Section 49-4 The Heat Capacity of Solids

19. In terms of the Einstein temperature \( T_E \), at what temperature will the molar internal energy of a solid achieve one-half its classical value of \( 3RT \)?

20. (a) Show that the molar internal energy \( E_{\text{int}} \) of a solid can be written, according to Einstein's theory of heat capacities, as

\[
E_{\text{int}} = 3RT_E \left( \frac{1}{e^{x} - 1} \right),
\]

in which \( x = T_E/T \), where \( T_E \) is the Einstein temperature

\[
hv/k. \] (b) Verify that \( E_{\text{int}} \) approaches its classical value of \( 3RT \) as \( T \to \infty \).

21. In terms of Einstein’s theory of heat capacity, (a) what is the molar heat capacity at constant volume of a solid at its Einstein temperature? Express your answer as a percentage of its classical value of 3R. (b) What is the molar internal energy at the Einstein temperature? Express your answer as a percentage of its classical value of 3RT.

22. Show that, at high enough temperatures, Einstein’s expression for the heat capacity of a solid, Eq. 14, reduces to the classical formula, Eq. 11.

23. The Einstein temperatures of lead, aluminum, and beryllium may be taken as 68 K, 290 K, and 690 K, respectively. For each of these elements, find (a) the frequency \( v \) of its atomic oscillators, (b) the spacing \( \Delta E \) between adjacent oscillator levels, and (c) the effective spring constant \( k \).

24. The Einstein temperature of aluminum may be taken as 290 K. According to Einstein’s theory of heat capacity, what are (a) its molar internal energy (see Problem 20) at 150 K and (b) its molar heat capacity, under constant-volume conditions, at 150 K?

25. A 12.0-g block of aluminum is heated from 80 K up to 180 K, under constant-volume conditions. How much heat is required according to (a) the classical theory of heat capacity and (b) Einstein’s quantum theory of heat capacity? The Einstein temperature for aluminum may be taken to be 290 K.

26. Assume that 25.0 g of aluminum at 80.0 K are mixed thoroughly with 12.0 g of aluminum at 200 K in an insulated container. What is the final temperature of the mixture? Assume that Einstein’s theory of heat capacities is valid and that, at these relatively low temperatures, the differences between the heat capacity at constant volume and that at constant pressure may be neglected. Assume further that there are no energy exchanges between the two aluminum specimens and the container. The Einstein temperature of aluminum may be taken to be 290 K.

Section 49-6 Einstein's Photon Theory

27. (a) By using the “best” values of the fundamental constants, as found in Appendix B, show that the energy \( E \) of a photon is related to its wavelength \( \lambda \) by

\[
E = \frac{1240 \text{ eV} \cdot \text{nm}}{\lambda}.
\]

This result can be useful in solving many problems. (b) The orange-colored light from a highway sodium lamp has a wavelength of 589 nm. How much energy is possessed by an individual photon from such a lamp?

28. Consider monochromatic light falling on a photographic film. The incident photons will be recorded if they have enough energy to dissociate a AgBr molecule in the film. The minimum energy required to do this is about 0.60 eV. Find the cutoff wavelength greater than which the light will not be recorded. In what region of the spectrum does this wavelength fall?

29. An atom absorbs a photon having a wavelength of 375 nm and immediately emits another photon having a wavelength
of 580 nm. What was the net energy absorbed by the atom in this process?

30. (a) A spectral emission line of hydrogen, important in radioastronomy, has a wavelength of 21.11 cm. What is its corresponding photon energy? (b) At one time the meter was defined as 1,650,763,733 wavelengths of the orange light emitted by a light source containing krypton-86 atoms. What is the corresponding photon energy of this radiation?

31. Most gaseous ionization processes require energy changes of $1.0 \times 10^{-18}$ to $1.0 \times 10^{-16}$ J. What region then of the Sun’s electromagnetic spectrum is chiefly responsible for creating the icosphere in the Earth’s atmosphere?

32. Under ideal conditions the normal human eye will record a visual sensation at 540 nm if incident photons are absorbed at a rate as low as 100 s$^{-1}$. To what power level does this correspond?

33. You wish to pick a substance for a photocell operable with visible light. Which of the following will do (work function in parentheses): tantalum (4.2 eV), tungsten (4.5 eV), aluminum (4.2 eV), barium (2.5 eV), lithium (2.3 eV), cesium (1.9 eV)?

34. Satellites and spacecrafts orbiting the Earth can become charged due, in part, to the loss of electrons caused by the photoelectric effect induced by sunlight on the space vehicle’s outer surface. Suppose that a satellite is coated with platinum, a metal with one of the largest work functions: $\phi = 5.32$ eV. Find the smallest-frequency photon that can eject a photoelectron from platinum. (Satellites must be designed to minimize such charging.)

35. (a) The energy needed to remove an electron from metallic sodium is 2.28 eV. Does sodium show a photoelectric effect for red light, with $\lambda = 678$ nm? (b) What is the cutoff wavelength for photoelectric emission from sodium and to what color does this wavelength correspond?

36. Find the maximum kinetic energy in eV of photoelectrons if the work function of the material is 2.33 eV and the frequency of the radiation is 3.19 $\times$ 10$^{15}$ Hz.

37. Incident photons strike a sodium surface having a work function of 2.28 eV, causing photoelectric emission. When a stopping potential of 4.92 V is imposed, there is no photocurrent. Find the wavelength of the incident photons.

38. Light of wavelength 200 nm falls on an aluminum surface. In aluminum, 4.2 eV is required to remove an electron. What is the kinetic energy of (a) the fastest and (b) the slowest emitted photoelectrons? (c) Find the stopping potential. (d) Calculate the cutoff wavelength for aluminum.

39. (a) If the work function for a metal is 1.85 eV, what would be the stopping potential for light having a wavelength of 410 nm? (b) What would be the maximum speed of the emitted photoelectrons at the metal’s surface?

40. The stopping potential for photoelectrons emitted from a surface illuminated by light of wavelength 491 nm is 710 mV. When the incident wavelength is changed to a new value, the stopping potential is found to be 1.43 V. (a) What is this new wavelength? (b) What is the work function for the surface?

41. Millikan’s photoelectric data for lithium are:

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>433.9</th>
<th>404.7</th>
<th>365.0</th>
<th>312.5</th>
<th>253.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stopping potential (V)</td>
<td>0.55</td>
<td>0.73</td>
<td>1.09</td>
<td>1.67</td>
<td>2.57</td>
</tr>
</tbody>
</table>

Make a plot like Fig. 11, which is for sodium, and find (a) the Planck constant and (b) the work function for lithium.

42. A lithium surface for which the work function is 2.49 eV is irradiated with light of frequency $6.33 \times 10^{14}$ Hz. The loss of electrons causes the metal to acquire a positive potential. What must this potential have become by the time its value prevents further loss of electrons from the surface?

43. A satellite in Earth orbit maintains a panel of solar cells at right angles to the direction of the Sun’s rays. Assume that the solar radiation is monochromatic with a wavelength of 550 nm and arrives at the rate of 1.38 kW/m$^2$. What must be the area of the panels in order that “one mole of photons” arrives each minute?

44. In the photon picture of radiation, show that if two parallel beams of light of different wavelengths are to have the same intensity, then the rates per unit area at which photons pass through any cross section of the beams are in the same ratio as the wavelengths.

45. An ultraviolet light bulb, emitting at 400 nm, and an infrared light bulb, emitting at 700 nm, each are rated at 130 W. (a) Which bulb radiates photons at the greater rate? (b) How many more photons does it generate per second than does the other bulb?

46. To remove an inner, most tightly bound, electron from an atom of molybdenum requires an energy of 20 keV. If this is to be done by allowing a photon to strike the atom, (a) what must be the associated wavelength of the photon? (b) In what region of the spectrum does the photon lie? (c) Could this process be called a photoelectric effect? Discuss your answers.

47. X rays with a wavelength of 710 pm eject photoelectrons from a gold foil, the electrons originating from deep within the gold atoms. The ejected electrons move in circular paths of radius $r$ in a region of uniform magnetic field $B$. Experiment shows that $rB = 188 \mu T \cdot m$. Find (a) the maximum kinetic energy of the photoelectrons and (b) the work done in removing the electrons from the gold atoms that make up the foil.

48. A special kind of light bulb emits monochromatic light at a wavelength of 630 nm. It is rated at 70.0 W and is 93.2% efficient in converting electrical energy to light. How many photons will the bulb emit over its 730-h lifetime?

49. Assume that a 100-W sodium-vapor lamp radiates its energy uniformly in all directions in the form of photons with an associated wavelength of 589 nm. (a) At what rate are photons emitted from the lamp? (b) At what distance from the lamp will the average flux of photons be 1.00 photon/(cm$^2 \cdot s$)? (c) At what distance from the lamp will the average density of photons be 1.00 photon/cm$^2$? (d) Calculate the photon flux and the photon density 2.00 m from the lamp.

50. Show, by analyzing a collision between a photon and a free electron (using relativistic mechanics), that it is impossible for a photon to give all its energy to the free electron. In other words, the photoelectric effect cannot occur for completely
free electrons; the electrons must be bound in a solid or in an atom.

Section 49-7 The Compton Effect

51. A particular x-ray photon has a wavelength of 41.6 pm. Calculate the photon's (a) energy, (b) frequency, and (c) momentum.

52. Find (a) the frequency, (b) the wavelength, and (c) the momentum of a photon whose energy equals the rest energy of the electron.

53. By how much does a sodium atom slow down upon absorbing a photon of wavelength 589 nm with which it collides head-on?

54. The quantity $h/mc$ in Eq. 25 is often called the Compton wavelength, $\lambda_c$, of the scattering particle and that equation is written

\[ \Delta \lambda = \lambda_c (1 - \cos \phi). \]

(a) Calculate the Compton wavelength of an electron. Of a proton. (b) What is the energy of a photon whose wavelength is equal to the Compton wavelength of the electron? Of the proton? (c) Show that in general the energy of a photon whose wavelength is equal to the Compton wavelength of a particle is just the rest energy of that particle.

55. Photons of wavelength 2.17 pm are incident on free electrons. (a) Find the wavelength of a photon that is scattered 35.0° from the incident direction. (b) Do the same if the scattering angle is 115°.

56. A 511-keV gamma-ray photon is Compton-scattered from a free electron in an aluminum block. (a) What is the wavelength of the incident photon? (b) What is the wavelength of the scattered photon? (c) What is the energy of the scattered photon? Assume a scattering angle of 72.0°.

57. Show that $\Delta E/E$, the fractional loss of energy of a photon during a Compton collision, is given by

\[ \frac{\Delta E}{E} = \frac{hv'}{mc^2} (1 - \cos \phi). \]

58. What fractional increase in wavelength leads to a 75% loss of photon energy in a Compton collision with a free electron?

59. Find the maximum wavelength shift for a Compton collision between a photon and a free electron.

60. A 6.2-keV x-ray photon falling on a carbon block is scattered by a Compton collision and its frequency is shifted by 0.010%. (a) Through what angle is the photon scattered? (b) How much kinetic energy is imparted to the electron?

61. An x-ray photon of wavelength $\lambda = 9.77$ pm is backscattered by an electron ($\phi = 180°$). Determine (a) the change in wavelength of the photon, (b) the change in energy of the photon, and (c) the final kinetic energy of the electron.

62. Calculate the fractional change in photon energy for a Compton collision with $\phi$ in Fig. 14 equal to 90° for radiation in (a) the microwave range, with $\lambda = 3.00$ cm, (b) the visible range, with $\lambda = 500$ nm, (c) the x-ray range, with $\lambda = 0.10$ nm, and (d) the gamma-ray range, with $\lambda = 1.30$ pm. What are your conclusions about the importance of the Compton effect in these various regions of the electromagnetic spectrum, judged solely by the criterion of energy loss in a single Compton encounter?

63. Through what angle must a 215-keV photon be scattered by a free electron so that it loses 10.0% of its energy?

64. Carry out the necessary algebra to eliminate $v$ and $\theta$ from Eqs. 21, 23, and 24 to obtain the Compton shift relation, Eq. 25.

65. (a) Show that when a photon of energy $E$ scatters from a free electron, the maximum recoil kinetic energy of the electron is given by

\[ K_{\text{max}} = \frac{E^2}{E + mc^2/2}. \]

(b) Find the maximum kinetic energy of the Compton-scattered electrons knocked out of a thin copper foil by an incident beam of 17.5-keV x rays.