PROBLEMS

Section 47-1 Multiple Slits

1. A diffraction grating 21.5 mm wide has 6140 rulings. (a) Calculate the distance d between adjacent rulings. (b) At what angles will maximum-intensity beams occur if the incident radiation has a wavelength of 589 nm?

2. A diffraction grating 2.86 cm wide produces a deviation of 33.2° in the second order with light of wavelength 612 nm. Find the total number of rulings on the grating.

3. With light from a gaseous discharge tube incident normally on a grating with a distance 1.73 μm between adjacent slit centers, a green line appears with sharp maxima at measured transmission angles θ = ± 17.6°, 37.3°, -37.1°, 65.2°, and -65.0°. Compute the wavelength of the green line that best fits the data.

4. A narrow beam of monochromatic light strikes a grating at normal incidence and produces sharp maxima at the following angles from the normal: 6° 40', 13° 30', 20° 20', 35° 40'. No other maxima appear at any angle between 0° and 35° 40'. The separation between adjacent ruling centers in the grating is 5040 nm. Find the wavelength of light used.

5. Light of wavelength 600 nm is incident normally on a diffraction grating. Two adjacent principal maxima occur at sin θ = 0.20 and sin θ = 0.30. The fourth order is missing. (a) What is the separation between adjacent slits? (b) What is the smallest possible individual slit width? (c) Name all orders actually appearing on the screen with the values derived in (a) and (b).

6. A diffraction grating is made up of slits of width 310 nm with a 930-nm separation between centers. The grating is illuminated by monochromatic plane waves, λ = 615 nm, the angle of incidence being zero. (a) How many diffraction maxima are there? (b) Find the width of the spectral lines observed in first order if the grating has 1120 slits.

7. Derive an expression for the intensity pattern for a three-slit “grating”:

\[
i = \frac{1}{4} I_m (1 + 4 \cos \phi + 4 \cos^2 \phi),
\]

where

\[
\phi = \frac{2 \pi d \sin \theta}{\lambda}.
\]

Assume that a ≪ λ and be guided by the derivation of the corresponding double-slit formula (Eq. 17 of Chapter 46).

8. (a) Using the result of Problem 7, show that the halfwidth of the fringes for a three-slit diffraction pattern, assuming θ small enough so that sin θ = θ, is

\[
\Delta \theta = \frac{\lambda}{3.2d}.
\]

(b) Compare this with the expression derived for the two-slit pattern in Problem 25. Chapter 45 and show that these results support the conclusion that for a fixed slit spacing the interference maxima become sharper as the number of slits is increased.

9. (a) Using the result of Problem 7, show that a three-slit “grating” has only one secondary maximum. Find (b) its location and (c) its relative intensity.

10. A three-slit grating has separation d between adjacent slits. If the middle slit is covered up, will the halfwidth of the intensity maxima become broader or narrower and by what factor? See Problem 8.

11. A diffraction grating has a large number N of slits, each of width a. Let I_{max} denote the intensity at some principal maximum, and let I_k denote the intensity of the kth adjacent secondary maximum. (a) If k ≪ N, show from the phasor diagram that, approximately, \( I_k / I_{max} = 1/(k + 1)^2 \pi^2\). (Compare this with the single-slit formula.) (b) For those secondary maxima that lie roughly midway between two adjacent principal maxima, show that roughly \( I_k / I_{max} = 1/N^2 \). (c) Consider the central principal maximum and those adjacent secondary maxima for which k ≪ N. Show that this part of the diffraction pattern quantitatively resembles that for one single slit of width Na.

Section 47-2 Diffraction Gratings

12. A diffraction grating has 200 rulings/mm and a principal maximum is noted at θ = 28°. (a) What are the possible wavelengths of the incident visible light? (b) What colors are they?

13. A grating has 315 rulings/mm. For what wavelengths in the visible spectrum can fifth-order diffraction be observed?

14. Show that in a grating with alternately transparent and opaque strips of equal width, all the even orders (except m = 0) are absent.

15. Given a grating with 400 rulings/mm, how many orders of the entire visible spectrum (400 – 700 nm) can be produced?

16. Assume that light is incident on a grating at an angle ψ as shown in Fig. 25. Show that the condition for a diffraction maximum is

\[
d(\sin \psi + \sin \theta) = m\lambda, \quad m = 0, 1, 2, \ldots
\]

Only the special case ν = 0 has been treated in this chapter (compare with Eq. 1).

Figure 25 Problem 16.

17. A transmission grating with d = 1.50 μm is illuminated at various angles of incidence by light of wavelength 600 nm. Plot as a function of angle of incidence (0 to 90°) the angular deviation of the first-order diffracted beam from the incident direction. See Problem 16.
18. Assume that the limits of the visible spectrum are arbitrarily chosen as 430 and 680 nm. Calculate the number of rulings per mm of a grating that will spread the first-order spectrum through an angular range of 20.0°.

19. White light (400 nm < λ < 700 nm) is incident on a grating. Show that, no matter what the value of the grating spacing d, the second- and third-order spectra overlap.

20. A grating has 350 rulings/mm and is illuminated at normal incidence by white light. A spectrum is formed on a screen 30 cm from the grating. If a 10-mm square hole is cut in the screen, its inner edge being 50 mm from the central maximum and parallel to it, what range of wavelengths passes through the hole?

Section 47-3 Dispersion and Resolving Power

21. The "sodium doublet" in the spectrum of sodium is a pair of lines with wavelengths 589.0 and 589.6 nm. Calculate the minimum number of rulings in a grating needed to resolve this doublet in the second-order spectrum.

22. A grating has 620 rulings/mm and is 5.05 mm wide. (a) What is the smallest wavelength interval that can be resolved in the third order at λ = 481 nm? (b) How many higher orders can be seen?

23. A source containing a mixture of hydrogen and deuterium atoms emits light containing two closely spaced red colors at λ = 656.3 nm whose separation is 0.180 nm. Find the minimum number of rulings needed in a diffraction grating that can resolve these lines in the first order.

24. (a) How many rulings must a 4.15-cm-wide diffraction grating have to resolve the wavelengths 415.496 nm and 415.487 nm in the second order? (b) At what angle are the maxima found?

25. In a particular grating the sodium doublet (see Problem 21) is viewed in third order at 10.2° to the normal and is barely resolved. Find (a) the ruling spacing and (b) the total width of the grating.

26. Show that the dispersion of a grating can be written

\[ D = \frac{\tan \theta}{\lambda} \]

27. A grating has 40,000 rulings spread over 76 mm. (a) What is its expected dispersion D in °/nm for sodium light (λ = 589 nm) in the first three orders? (b) What is its resolving power in these orders?

28. Light containing a mixture of two wavelengths, 500 nm and 600 nm, is incident normally on a diffraction grating. It is desired (1) that the first and second principal maxima for each wavelength appear at θ = 30°, (2) that the dispersion be as high as possible, and (3) that the third order for 600 nm be a missing order. (a) What should be the separation between adjacent slits? (b) What is the smallest possible individual slit width? (c) Name all orders for 600 nm that actually appear on the screen with the values derived in (a) and (b).

29. A diffraction grating has a resolving power \( R = \frac{\lambda}{\Delta \lambda} = N \). (a) Show that the corresponding frequency range \( \Delta \nu \) that can just be resolved is given by \( \Delta \nu = c/N\lambda \).

30. (b) From Fig. 1, show that the "times of flight" of the two extreme rays differ by an amount \( \Delta t = \frac{(Nd/c) \sin \theta}{c} \). (c) Show that \( (\Delta \nu)(\Delta t) = 1 \), this relation being independent of the various grating parameters. Assume \( N \gg 1 \).

Section 47-4 X-Ray Diffraction

30. X rays of wavelength 0.122 nm are found to reflect in the second order from the face of a lithium fluoride crystal at a Bragg angle of 28.1°. Calculate the distance between adjacent crystal planes.

31. A beam of x rays of wavelength 29.3 pm is incident on a calcite crystal of lattice spacing 0.313 nm. Find the smallest angle between the crystal planes and the beam that will result in constructive reflection of the x rays.

32. Monochromatic high-energy x rays are incident on a crystal. If first-order reflection is observed at Bragg angle 3.40°, at what angle would second-order reflection be expected?

33. An x-ray beam, containing radiation of two distinct wavelengths, is scattered from a crystal, yielding the intensity spectrum shown in Fig. 26. The interplanar spacing of the scattering planes is 0.94 nm. Determine the wavelengths of the x rays present in the beam.

Figure 26  Problem 33.

34. In comparing the wavelengths of two monochromatic x-ray lines, it is noted that line A gives a first-order reflection maximum at a glancing angle of 23.2° to the face of a crystal. Line B, known to have a wavelength of 96.7 pm, gives a third-order reflection maximum at an angle of 58.0° from the same face of the same crystal. (a) Calculate the interplanar spacing. (b) Find the wavelength of line A.

35. Monochromatic x rays are incident on a set of NaCl crystal planes whose interplanar spacing is 39.8 pm. When the beam is rotated 51.3° from the normal, first-order Bragg reflection is observed. Find the wavelength of the x rays.

36. Show that, in Bragg diffraction by a monochromatic beam of x rays, no intense maxima will be obtained if the wavelength of the x rays is greater than twice the largest crystal plane separation. See Question 20.

37. Prove that it is not possible to determine both wavelength of radiation and spacing of Bragg reflecting planes in a crystal by measuring the angles for Bragg reflection in several orders.

38. Assume that the incident x-ray beam in Fig. 27 is not monochromatic but contains wavelengths in a band from 95.0 to 139 pm. Will diffracted beams, associated with the planes
shown, occur? If so, what wavelengths are diffracted? Assume \( d = 275 \) pm.

39. First-order Bragg scattering from a certain crystal occurs at an angle of incidence of 63.8°; see Fig. 28. The wavelength of the x rays is 0.261 nm. Assuming that the scattering is from the dashed planes shown, find the unit cell size \( a_0 \).

40. Monochromatic x rays (\( \lambda = 0.125 \) nm) fall on a crystal of sodium chloride, making an angle of 42.2° with the reference line shown in Fig. 27. The planes shown are those of Fig. 18a, for which \( d = 0.252 \) nm. Through what angles must the crystal be turned to give a diffracted beam associated with the planes shown? Assume that the crystal is turned about an axis that is perpendicular to the plane of the page.

41. Consider an infinite two-dimensional square lattice as in Fig. 16b. One interplanar spacing is obviously \( a_0 \) itself. (a) Calculate the next five smaller interplanar spacings by sketching figures similar to Fig. 18a. (b) Show that the general formula is

\[
d = a_0 \sqrt{h^2 + k^2},
\]

where \( h \) and \( k \) are both relatively prime integers that have no common factors other than unity.

42. In Sample Problem 5 the \( m = 1 \) beam, permitted by interference considerations, has zero intensity because of the diffracting properties of the unit cell for this geometry of beams and crystal. Prove this. (Hint: Show that the “reflection” from an atomic plane through the top of a layer of unit cells is canceled by a “reflection” from a plane through the middle of this layer of cells. All odd-order beams prove to have zero intensity.)