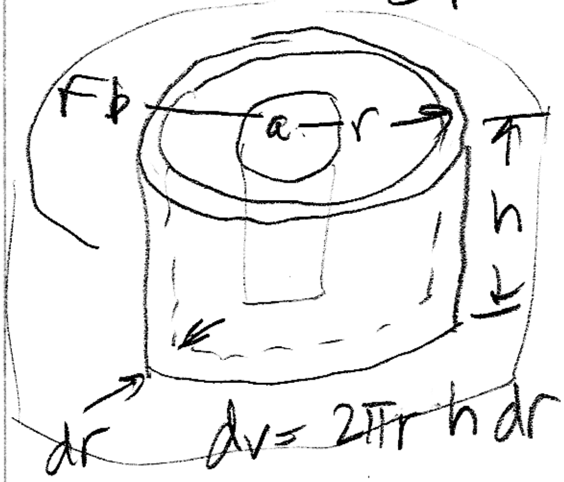
Can be thought of as residing in the field itself...

<u>energy density</u> $= \frac{1}{8\pi} B^2$

Toroid : $L = \frac{2N^2 h}{c^2} \ln\left(\frac{b}{a}\right)$

$$U = \frac{N^2 h}{c^2} \ln\left(\frac{b}{a}\right) I_0^{\,2}$$

$$B = \frac{2NI_0}{cr} \qquad B^2 = \frac{4N^2 I_0^{\,2}}{c^2 r^2}$$



$$U = \frac{1}{8\pi}\int_a^b B^2 dv$$

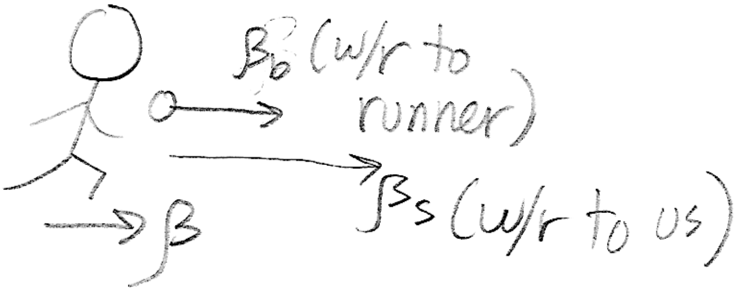
$$= \frac{1}{8\pi} \cdot \int_a^b \frac{4N^2 I_0^{\,2}}{c^2 r^2} 2\pi r h dr$$

$$= \frac{N^2 h}{c^2} I_0^{\,2} \int_a^b \frac{dr}{r}$$

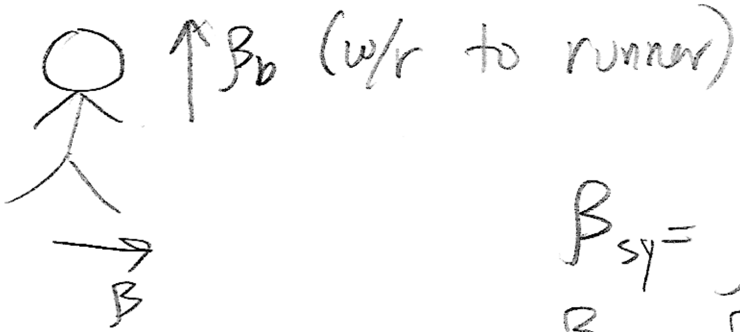
$$U = \frac{N^2 h}{c^2} I_0^{\,2} \ln\left(\frac{b}{a}\right)$$

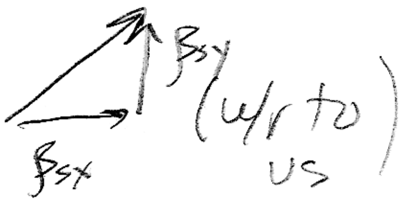
Review

- Addition of velocities



$$\beta_s = \frac{\beta + \beta_s}{1 + \beta\beta_s}$$





$$\beta_{sy} = \beta_b \cdot \sqrt{1-\beta^2}$$

$$\beta_{sx} = \beta$$

- Relativistic Mass, Momentum:

$$m(\beta) = \frac{m_0}{\sqrt{1-\beta^2}} = m_0 \cdot \gamma$$

$$E = m(\beta)c^2 = m_0 \gamma c^2$$

- 4 vectors:

$$\left(t, \frac{x}{c}, \frac{y}{c}, \frac{z}{c}\right)$$

$$\left(\frac{E}{c^2}, \frac{p_x}{c}, \frac{p_y}{c}, \frac{p_z}{c}\right)$$

$$p_x' = \gamma\left(p_x - \beta \frac{E'}{c}\right)$$

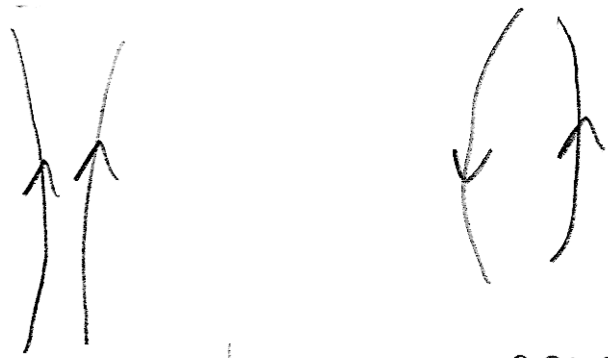
$$p_y' = p_y$$

$$p_z' = p_z$$

$$E' = \gamma(E - \beta c p_x)$$

motion in x-direction.

Magnetism

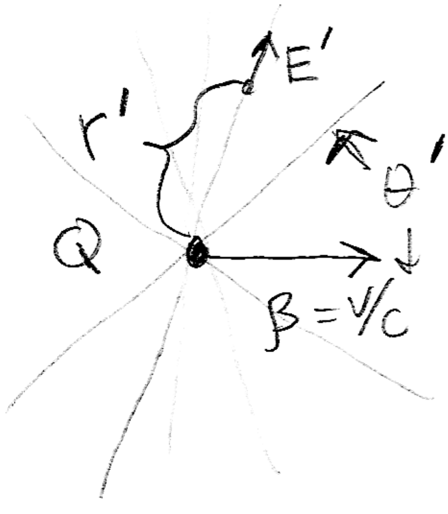


like currents attract

opposite repel

$$\vec{F} = q\vec{E} + q\left(\frac{\vec{v}}{c} \times \vec{B}\right)$$

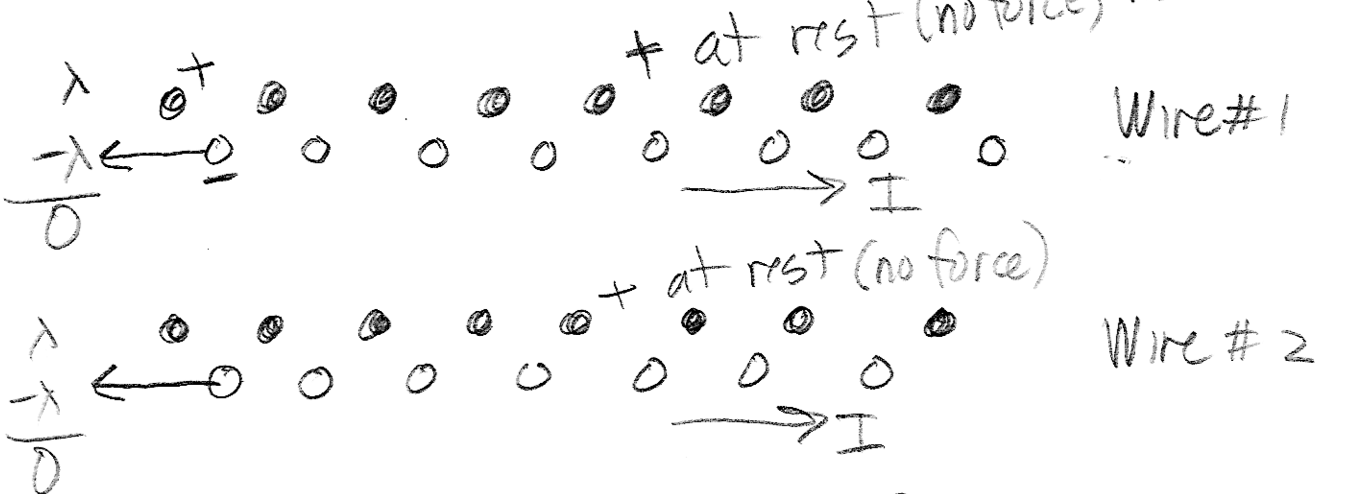
<u>Charge</u> is invariant w/r to Lorentz transformations



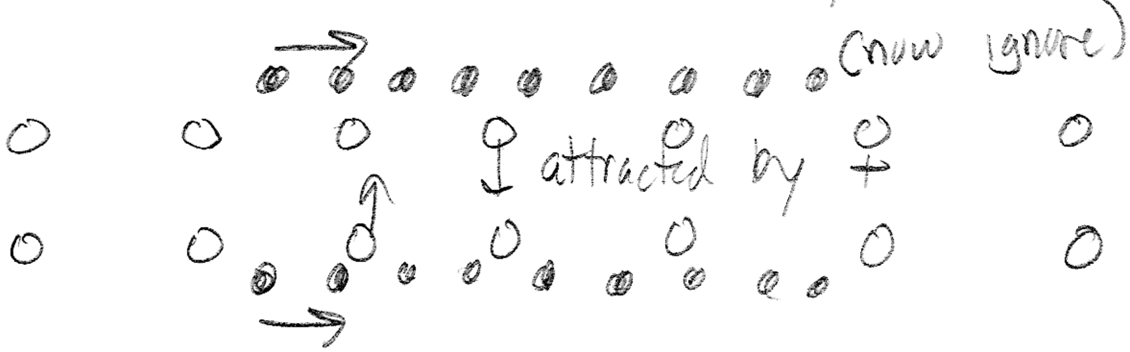
$$E' = \frac{Q}{r'^2} \cdot \frac{1-\beta^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$

Even though $\vec{E}' \neq \vec{E}$ in different frames, $\vec{F}' = q\vec{E}'$ $\vec{F} = q\vec{E}$ for ELECTRIC portion of force. Is TOTAL force only in frame where charge is at rest!

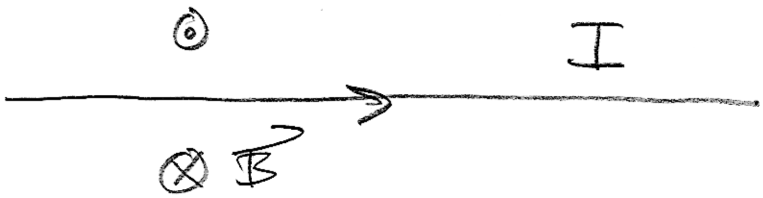
LAB frame:

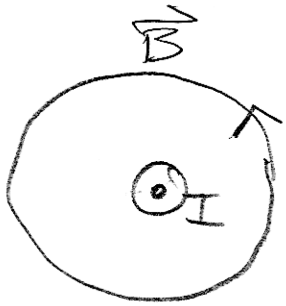


Cannot get total force on − in this frame.. must boost to its frame.. + charge density increases
− charge density decreases.



Wire's $\vec{B}$ field:





Thumb → along I
Fingers → along $\vec{B}$

$$|\vec{B}| = \frac{2I}{rc}$$

I: esu/s
r: cm
c: cm/s

Gauss,
10^4 Gauss = 1 Tesla (MKS)

MKS $|\vec{B}| = \frac{\mu_0}{2\pi} \frac{I}{r}$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$

I → amps, r = m

Two Wires: Force/length

$$\frac{F}{l} = 2 \frac{I_1 I_2}{d c^2} \quad \text{cgs}$$

$$= \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} \quad \text{MKS.}$$

$(= \mu_0 I)$

$$\oint \vec{B} \cdot d\vec{s} = \frac{4\pi}{c} \cdot I$$

around loop

I pokes through loop

$$\text{curl } \vec{B} = \frac{4\pi}{c} \vec{J}$$

$$\text{div } \vec{B} = 0$$

$$\vec{B} = \text{curl } \vec{A}$$

$\vec{A}$: • vecter potential
• tends to parallel I, but falls as $1/r$

Brot Savart

$$d\vec{B} = \frac{I \, d\vec{l} \times \hat{r}}{c r^2}$$

$d\vec{l}$: small length of wire, carrying current I

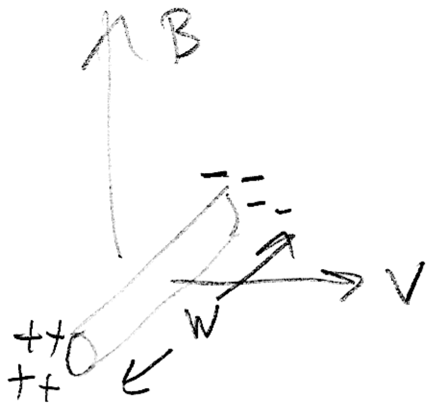
field of ring of current, coil of current.

Field Transformation

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} \qquad \vec{B}'_{\parallel} = \vec{B}_{\parallel}$$

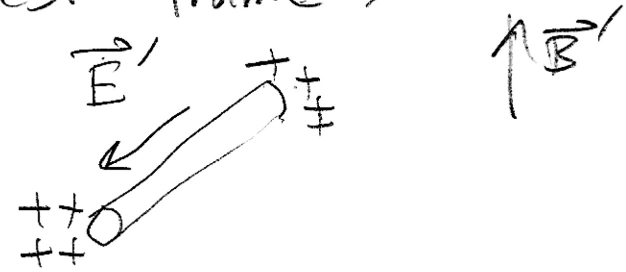
$$\vec{E}'_{\perp} = \gamma(\vec{E}_{\perp} + \vec{\beta}\times\vec{B}_{\perp}) \qquad \vec{B}'_{\perp} = \gamma(\vec{B}_{\perp} - \vec{\beta}\times\vec{E}_{\perp})$$

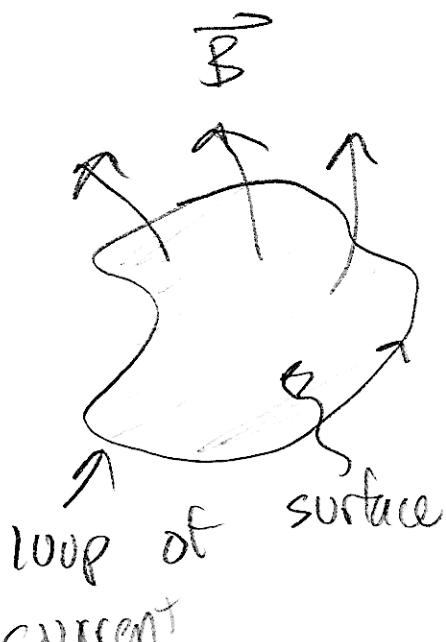
Induction



$$\text{voltage} = \frac{v}{c}\cdot B\cdot w$$

in rest frame:





$$\mathcal{E} = -\frac{1}{c}\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int_{\text{surface}} \vec{B}\cdot d\vec{a}$$

$$\mathcal{E} = \oint_{\text{circuit}} \vec{E}\cdot d\vec{s} \rightarrow \text{"voltage"}$$

$\frac{d\Phi_B}{dt}$: circuit can move $\left.\right\}$ equiv.
$\vec{B}$ can change

Lenz's Law

- Induced current contributes increment to $\vec{B}$ that tends to cancel $d\Phi_B/dt$.

- Induced current "lags" inducing current by $\pi/2$ radians.

Mutual Inductance

$$\mathcal{E}_{21} = -M_{21} \frac{dI_1}{dt}$$

Self

$$\mathcal{E} = -L \frac{dI}{dt}$$

L-R circuits.

$\vec{B}$'s energy.