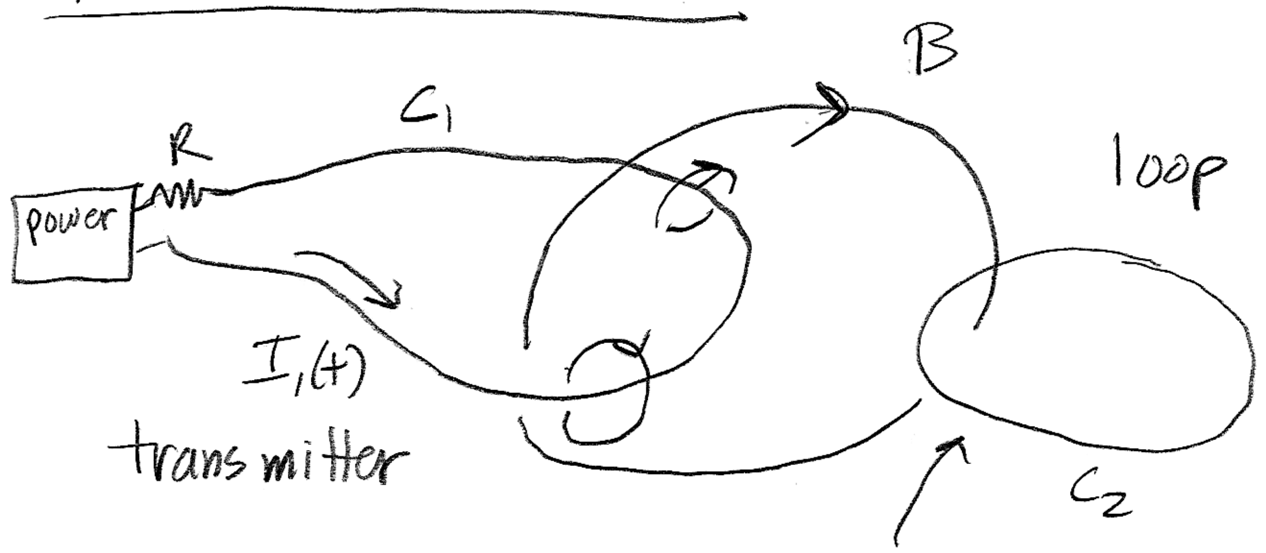
Mutual Inductance



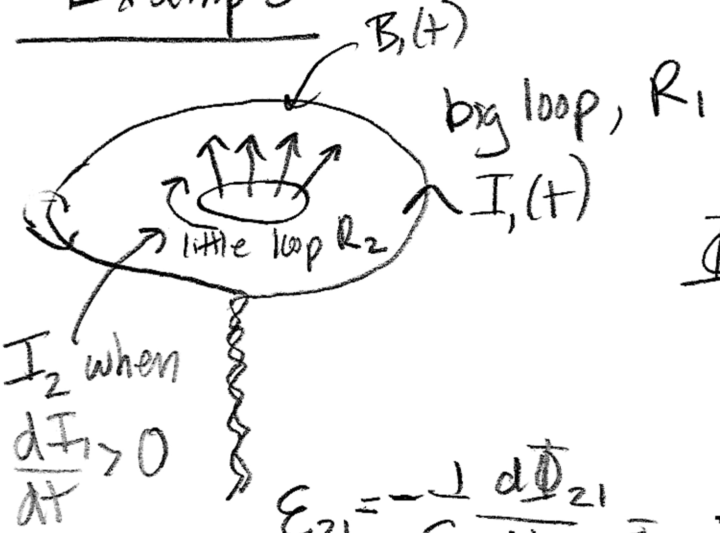
$$\Phi_{21}(t) \propto I_1$$

$$\mathcal{E}_{21} \propto \frac{d\Phi_{21}}{dt} \propto \frac{dI_1}{dt}$$

$$\mathcal{E}_{21} = -\underbrace{M_{21}}\frac{dI_1}{dt}$$

defines the mutual inductance.

Example



$$B_1 = \frac{2\pi}{c}\frac{I_1}{R_1}$$

$$\Phi_{21} \cong (\pi R_2^2)\, B_1$$

$$= \frac{2\pi^2 R_2^2}{c\, R_1} I_1$$

$$\mathcal{E}_{21} = -\frac{1}{c}\frac{d\Phi_{21}}{dt} = -\frac{2\pi^2}{c^2}\frac{R_2^2}{R_1}\frac{dI_1}{dt}$$

Wrap: N_1 loops around outside
N_2 loops around inside.

$$B_1 \propto N_1 \qquad \text{Area}_2 \propto N_2$$

$$M_{21} = \left(2\pi^2 \frac{R_2^2}{R_1}\right) N_1 N_2$$

And so...

$$M_{21} = \frac{2\pi^2}{c^2}\,\frac{R_2^{\,2}}{R_1}$$ ← kind of like capacitance, depends on geometry.

cgs: $\frac{s^2}{cm}$ (NEVER used!)

MKS: "henrys"... for R_1, R_2 in centimeters,

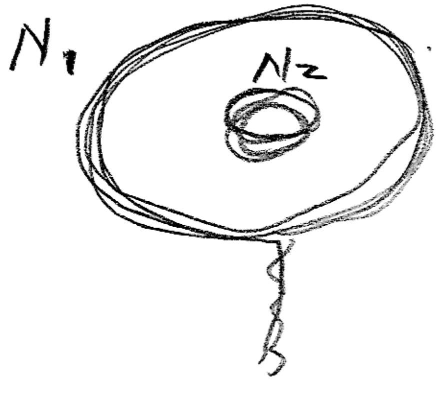
$$M_{21} = \left(2\pi^2 \cdot \frac{R_2^{\,2}}{R_1}\right) \cdot 10^{-9}\ \text{H}$$

↑ DROP the c^2!!

$R_2 = 15$ cm $\quad R_1 = 1$ cm

$$\boxed{M_{21} = 4.4 \cdot 10^{-6}\ \text{Henry}}$$

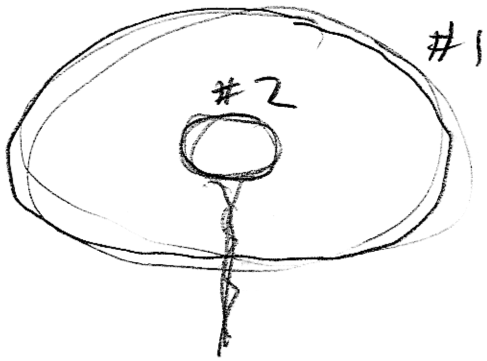
Wrap: N_1 loops around outside
N_2 loops around inside.



$B_1 \propto N_1 \qquad A_2^{\text{rea}} \propto N_2$

$$M_{21} = \left(2\pi^2\,\frac{R_2^{\,2}}{R_1}\right) N_1 N_2$$

Reciprocity

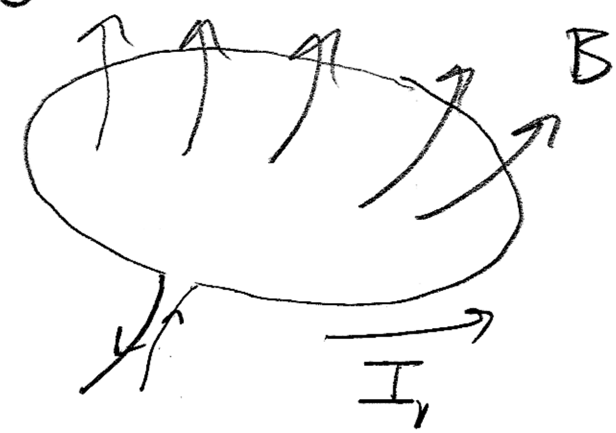


$M_{12} = M_{21}$!!!

proof: see book.

Self-Inductance (the big fun!)

Circuits put magnetic flux through themselves!

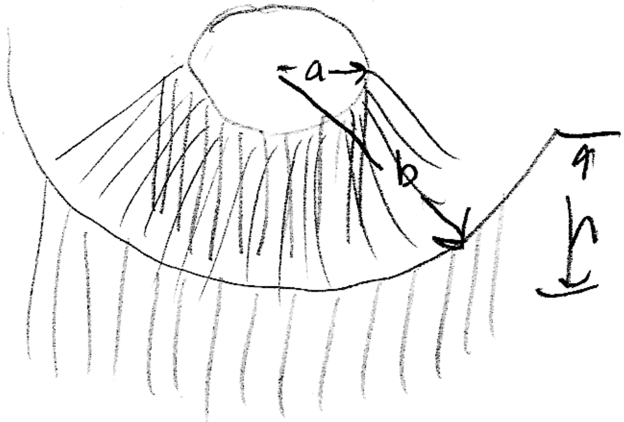


$$\mathcal{E}_{11} = -L_1 \frac{dI_1}{dt}$$

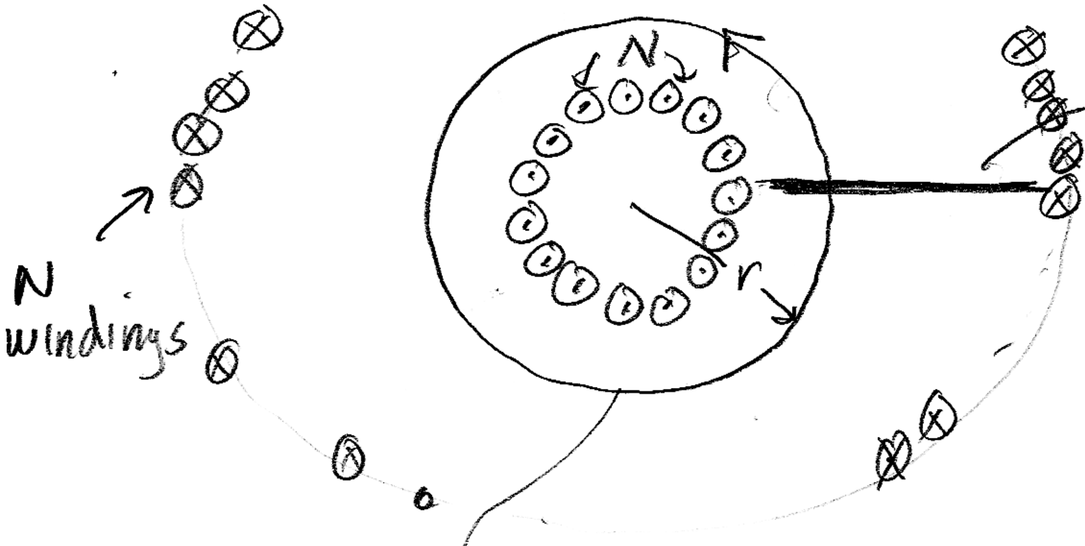
self inductance.

Physically, self inductance opposes huge changes in I_1 in short times,

Example: the toroid



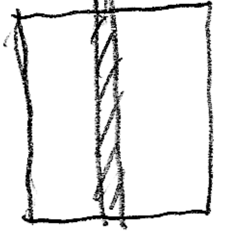
Cross sectional view:



$$2\pi r \cdot B = \frac{4\pi}{c} \cdot N \cdot I$$

$$B = \frac{2NI}{cr}$$

flux through this surface is



$$d\Phi_{B1} = \frac{2NI}{cr} \cdot h$$

$$\Phi_{B1} = \frac{2NIh}{c} \int_a^b \frac{dr}{r} = \frac{2NIh}{c} \ln\left(\frac{b}{a}\right)$$

There are N "loops" in the toroid.

$$\Phi_B = N\Phi_{B1} = \frac{2N^2 I h}{c} \ln\left(\frac{b}{a}\right)$$

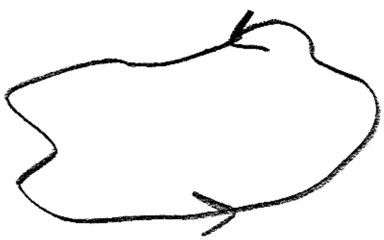
$$\mathcal{E} = -\frac{1}{c}\frac{d\Phi_B}{dt} = -\frac{2N^2 h}{c^2} \cdot \ln\left(\frac{b}{a}\right)\frac{dI}{dt}$$

$$L = \frac{2N^2 h}{c^2} \ln\left(\frac{b}{a}\right) \quad \text{cgs}$$

$$L(\text{henrys}) = (2 \cdot 10^{-9})\, N^2 h \ln\left(\frac{b}{a}\right)$$

$I \to$ amps
$h \to$ cm(!)

Turns out:
self inductance of a loop of wire.



is hard to compute!
(but not to measure).

Self inductance ... what inductors have!