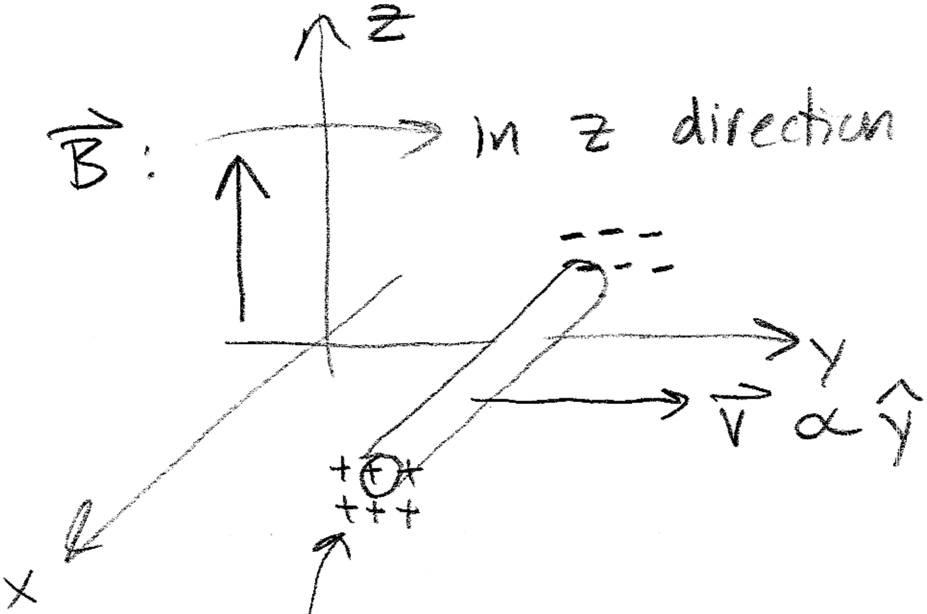
Conducting Bar Moving in a Uniform Magnetic Field.

$\vec{B}$: in z direction everywhere.



bar laying $\parallel$ to x

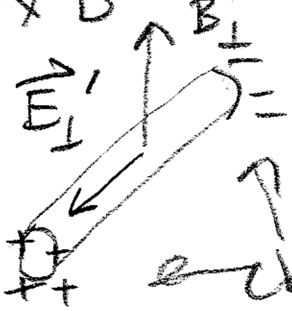
$\vec{v} \times \vec{B}$ is $\parallel$ to $+x$ ...

+ charges in wire get pushed toward $+x$, $-$ toward $-x$

movement of charge creates an $\vec{E}$ inside bar to cancel $\vec{v} \times \vec{B}$

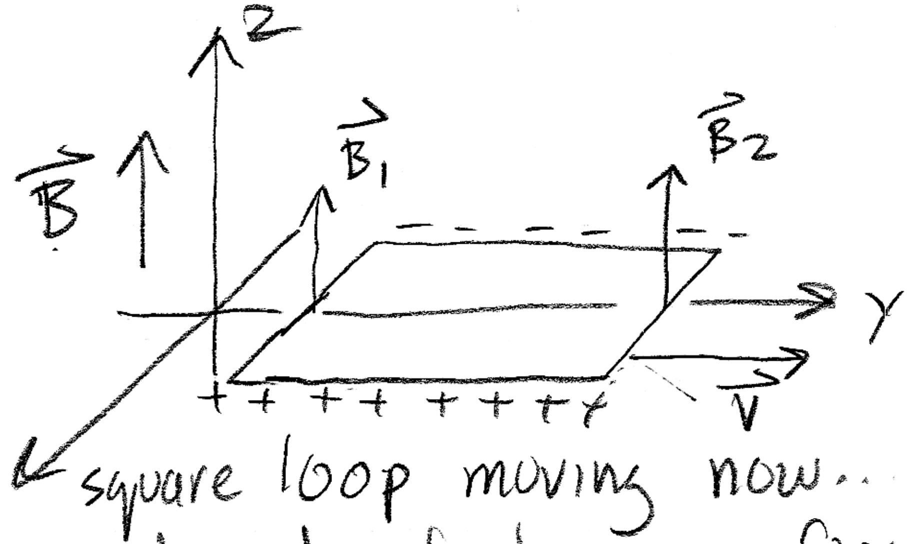
Boost to rest frame of bar...

$\vec{E}'_{\perp} = \gamma \vec{\beta} \times \vec{B}$



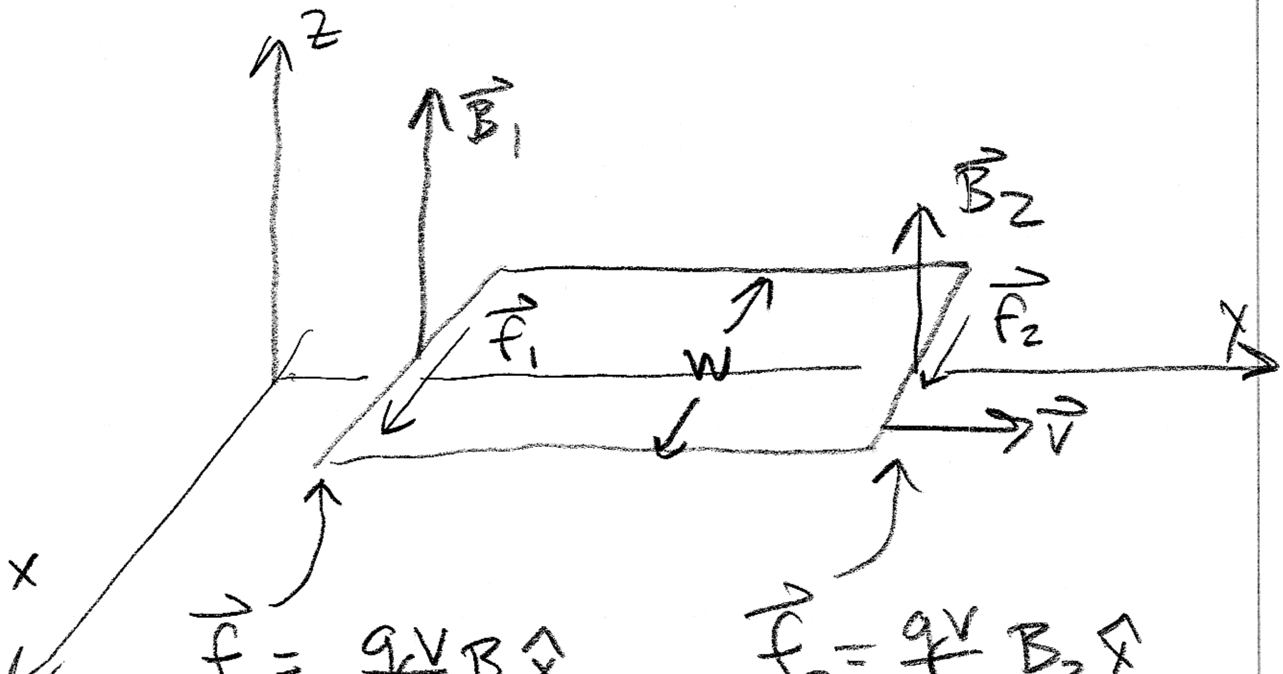
✓ in This frame, $\vec{v} = 0$, $\vec{v} \times \vec{B} = 0$

BUT $\vec{E}'$ field moves charge, $\vec{E}'_{total} = 0$ inside



square loop moving now.. both end rods feel same force, + charge migrates to one side, − to the other...

The real point: suppose $\vec{B}_1 \neq \vec{B}_2$



$\vec{f}_1 = \frac{qv}{c} B_1 \hat{x}$ $\quad$ $\vec{f}_2 = \frac{qv}{c} B_2 \hat{x}$

charge can be hurled forward at $\vec{B}_1$, gaining energy.. $\frac{qv}{c} B_1 w$, which is enough to overcome the barrier of $\frac{qv}{c} B_2 w$

Charge q can gain energy

$$= \frac{qv}{c}(B_1 - B_2)w \quad \text{per turn!}$$

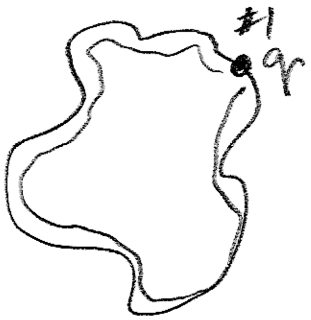
$$\equiv q \cdot \mathcal{E} \qquad \mathcal{E} = \text{electromotive}$$

force ... not a force at all.

$$\mathcal{E} = \frac{v}{c}(B_1 - B_2)w$$

$\Rightarrow$ related to rate of change of flux.

more generally,

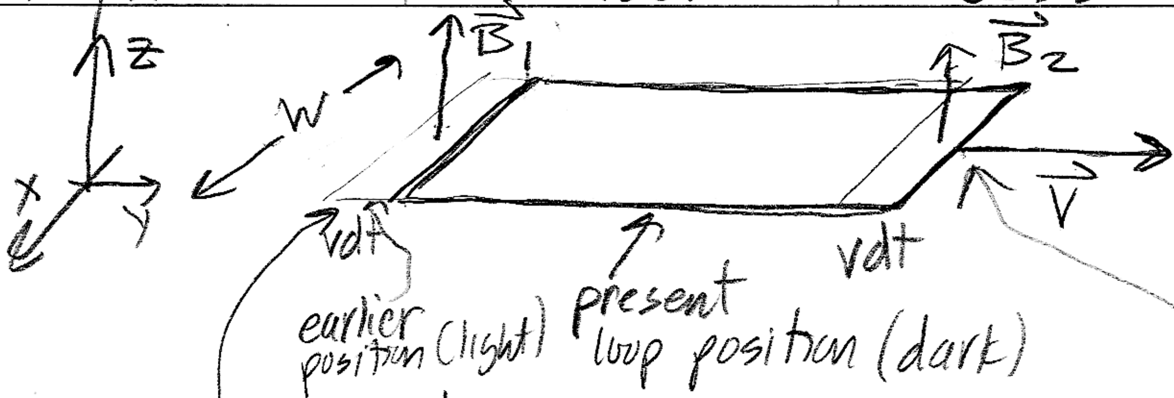


$$\mathcal{E} \equiv \frac{1}{q}\oint \vec{f} \cdot d\vec{s}$$

symbol was used before for the chemical work done.

$\mathcal{E}$ here is related to the rate of change of magnetic flux ...

look at $\mathcal{E} = \frac{w \cdot v}{c}(B_1 - B_2)$



when loop moves vdt, flux

$B_1 w v dt$ lost at back

$B_2 w v dt$ gained at front

$$d\Phi_B = \underbrace{(B_2 - B_1)}_{\text{field change}} \underbrace{wvdt}_{\text{area}}$$

so $$\boxed{\mathcal{E} = \frac{1}{c}\frac{-d\Phi_B}{dt} = (B_1 - B_2)\frac{wv}{c}}$$

what does the − sign really mean?

→ Current flows in the direction that creates a new, contributed $\vec{B}$ field that tends to counteract the change brought about by the motion (Lenz's Law)