$\boxed{\text{curl } \vec{E} = 0}$ why? no magnetic currents

Electric Charge $\rightarrow$ $\text{div } \vec{E} = 4\pi\rho$

Electric Current $\rightarrow$ $\text{curl } \vec{B} = \frac{4\pi}{c}\vec{J}$

no Magnetic Charge $\rightarrow$ $\text{div } \vec{B} = 0$

no Magnetic Current $\rightarrow$ $\text{curl } \vec{E} = 0$

$$\vec{E} = -\text{grad } \phi$$

$$\vec{B} = \left(\text{some diff operator}\right) \rightarrow \left(\text{some potential}\right)$$

$$\vec{B} = \text{curl } \vec{A}$$

called the "vector potential"

<u>recall</u>: $\nabla^2 \phi = -4\pi\rho \rightarrow \phi \propto \frac{1}{r}$

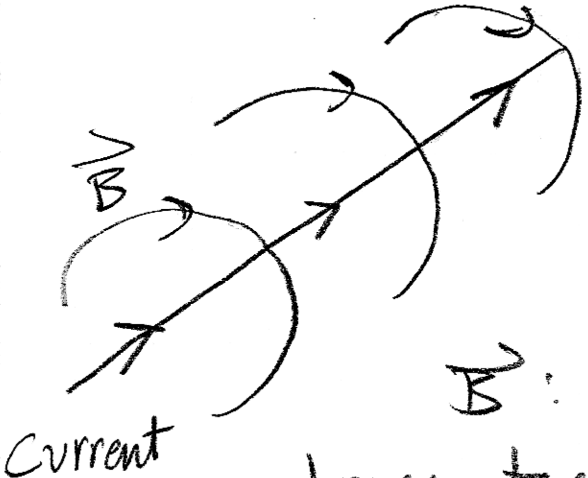
turns out

$$\left.\begin{aligned} \nabla^2 A_x &= -4\pi J_x \\ \nabla^2 A_y &= -4\pi J_y \\ \nabla^2 A_z &= -4\pi J_z \end{aligned}\right] \rightarrow A_i \propto \frac{1}{r}$$

from a portion of current.

Proof... see p. 221 of text

Let's work with it...

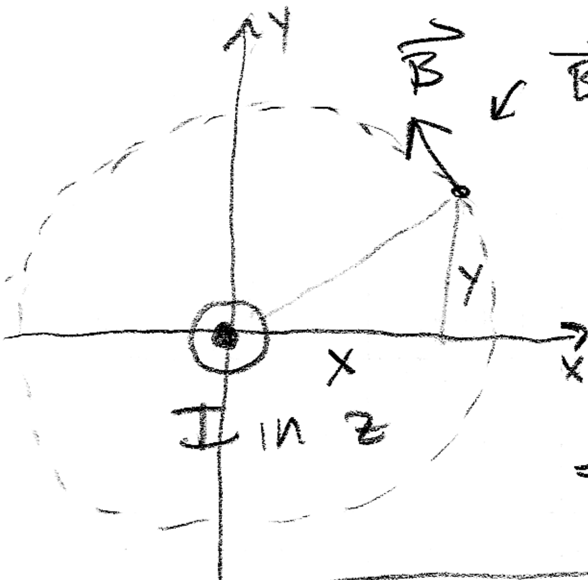


$$|\vec{B}| = \frac{2I}{rc} \Big] \ \infty \text{ long}$$

integrate

$$|\vec{A}| \propto \ln r \quad (\text{gets magnitude})$$

$\vec{B}$: get down to brass tacks.



$\vec{B} \propto -y\hat{x} + x\hat{y}$ .. $\perp$ to $x\hat{x} + y\hat{y}$

$$\vec{B} = \frac{2I}{cr} \cdot \left(\frac{-y}{\sqrt{x^2+y^2}}\hat{x} + \frac{x}{\sqrt{x^2+y^2}}\hat{y} \right)$$

$$= \frac{2I}{c} \left(\frac{-y\hat{x} + x\hat{y}}{x^2+y^2} \right)$$

$\boxed{\vec{A}: \text{ direction } \parallel \text{ to } I \ ..}$

$\propto \hat{z}$ $\qquad \propto \ln\sqrt{x^2+y^2} = \ln r$

try $\vec{A} = C \cdot \hat{z} \cdot \ln\sqrt{x^2+y^2}$

$$\text{curl}\,\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{x}\frac{\partial}{\partial y}A_z - \hat{y}\frac{\partial}{\partial x}A_z$$

$$\frac{\partial}{\partial y}A_z = C\frac{\partial}{\partial y}\ln\sqrt{x^2+y^2} = C\frac{1}{2}\frac{\partial}{\partial y}\ln(x^2+y^2)$$

$$= C\frac{1}{2}\cdot\frac{2y}{x^2+y^2} = \frac{y}{x^2+y^2}C$$

$$\frac{\partial}{\partial x}A_z = \frac{x}{x^2+y^2}C$$

$$\text{curl}\,\vec{A} = \frac{\hat{x}y - \hat{y}x}{x^2+y^2}\cdot C$$

$$= \frac{2I}{c}\left(\frac{-y\hat{x} + x\hat{y}}{x^2+y^2}\right)$$

$$C = -\frac{2I}{c}$$

$$\vec{A} = -\frac{2I}{c}\hat{z}\ln\sqrt{x^2+y^2} = -\frac{I}{c}\hat{z}\ln(x^2+y^2)$$

Qualitatively...

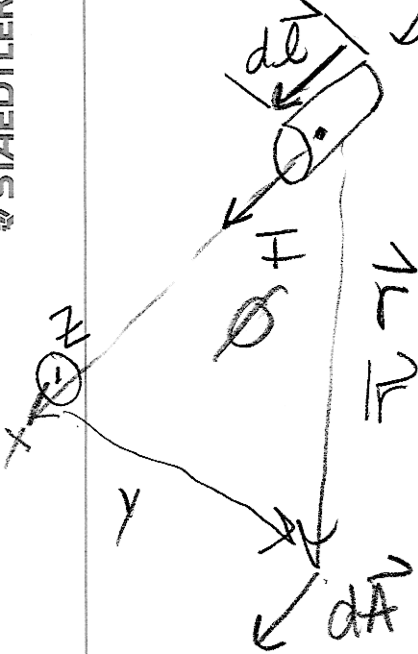
$\dfrac{I\hat{z}}{c} \rightarrow$ "source" of $\vec{A}$
like a "charge"

but, like potential, must account for $1/r$

short wire segment

direction of $I \rightarrow d\vec{l}$

$|d\vec{l}|$ = length of segment



$|\vec{r}| = \sqrt{x^2+y^2} = r$

$$d\vec{A} = \frac{I}{c}\,\frac{d\vec{l}}{r}$$

(integrate to get full $\vec{A}$)

call $d\vec{l} = dl\,\hat{x}$ (defines $\hat{x}$ direction)

$$d\vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & \cancel{A_y}^{\,0} & \cancel{A_z}^{\,0} \end{vmatrix}$$

$$= \hat{y}\,\frac{\partial A_x}{\partial z} - \hat{z}\,\frac{\partial A_x}{\partial y}$$

$$\frac{\partial A_x}{\partial z} = 0 \qquad \frac{\partial A_x}{\partial y} = \frac{I\,dl}{c}\,\frac{\partial}{\partial y}\,\frac{1}{\sqrt{x^2+y^2}}$$

$$= \frac{I\,dl}{c}\,\cdot -\frac{1}{2}\,\frac{2y}{(x^2+y^2)^{3/2}}$$

$$= -\frac{I\,dl}{c}\,\frac{\sin\phi}{r^2}$$

$$d\vec{B} = +\frac{I\,dl}{r^2}\,\hat{z}\sin\phi$$

$$\boxed{d\vec{B} = \frac{I\,d\vec{l}\times\hat{r}}{c\,r^2}}$$

similar to $d\vec{E} = \frac{q\,\hat{r}}{r^2}$

$$q \to \frac{I}{c}|d\vec{l}| \qquad \hat{r} \to \frac{d\vec{l}\times\hat{r}}{|d\vec{l}|}$$

"Biot–Savart Law"

$\approx$ "Coulomb's Law" for

$\vec{B}$ fields.