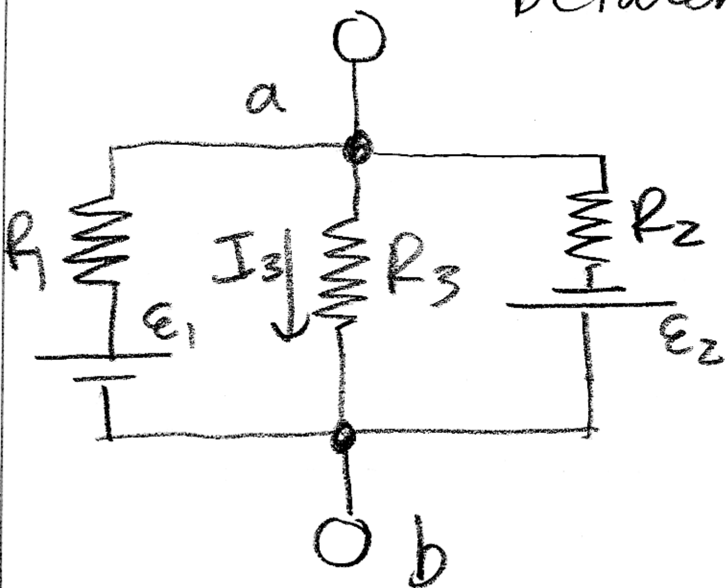
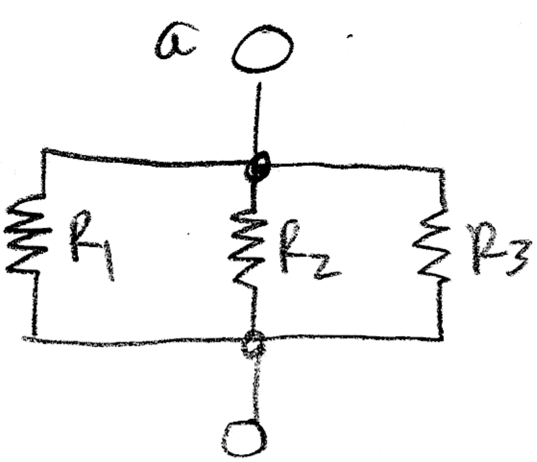
① $\mathcal{E}_{eq}$ = voltage you get between external terminals when nothing connected between terminals..



$$\mathcal{E}_{eq} = I_3 R_3$$

$$= \frac{R_3(\mathcal{E}_1 R_2 - \mathcal{E}_2 R_1)}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

② R_{eq} = Resistance between terminals when $\mathcal{E}_1 = \mathcal{E}_2 = 0$

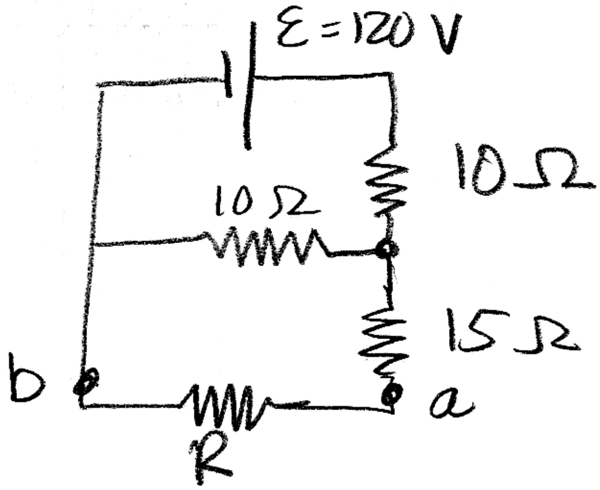


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

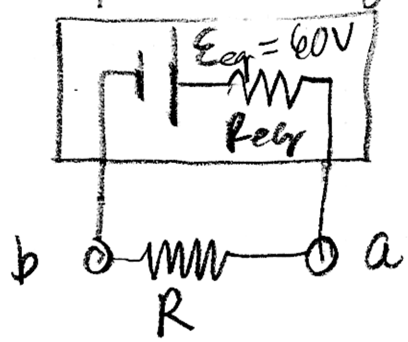
$$R_{eq} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

<u>Proof?</u> → <u>linearity</u>

Problem 4.22:

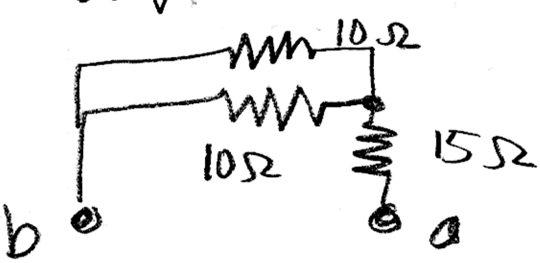


What value of R maximizes the power dissipated in R? Thévinin: $R = \infty$, only 2 $10\,\Omega$ to ground. $\mathcal{E}_{eq} = \frac{1}{2} \cdot 120\text{V} = 60\text{V}$

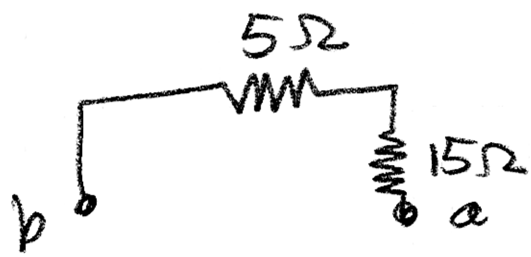


$$\mathcal{E}_{eq} = 60\text{V}$$

$R_{eq} \Rightarrow$



$$I = \frac{\mathcal{E}_{eq}}{(R + R_{eq})}$$



$$R_{eq} = 20\,\Omega$$

$$P = I^2 R = \frac{R}{(R + R_{eq})^2} \cdot \mathcal{E}_{eq}^2$$

in video, neglected one power of $\mathcal{E}_{eq}$!

$$\frac{dP}{dR} = \frac{1}{(R+R_{eq})^2} - \frac{2R}{(R+R_{eq})^3} = \frac{R_{eq} + R - 2R}{(R+R_{eq})^3} = 0$$

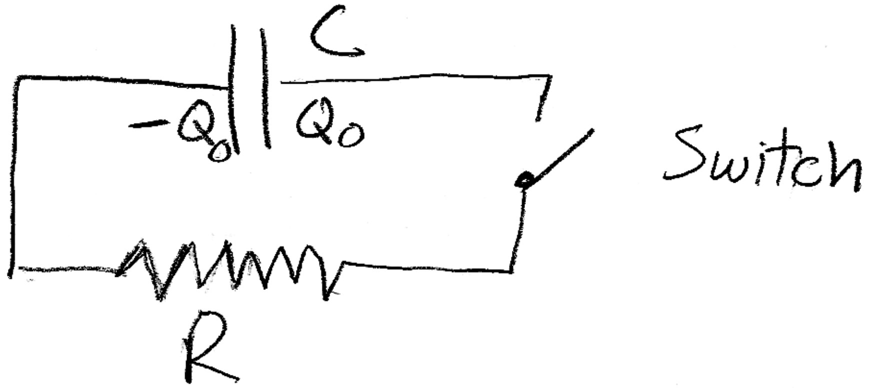
$$\boxed{\text{Then} \quad R = R_{eq} = 20\,\Omega}$$

$$P = \frac{20}{40} \cdot \frac{1}{40} \cdot 60^2$$

$$\boxed{P = \frac{3}{4} \cdot 60 = 45\text{ W}}$$

RC Circuits

A case where "steady state" is not the case.

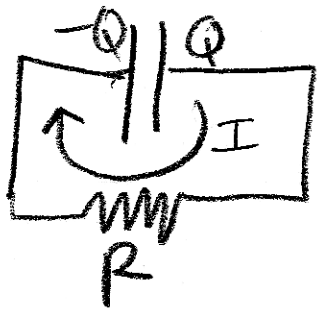


$-\infty < t < 0$: switch open

$Q_0, -Q_0$ on capacitor ... $V_0 = \frac{Q_0}{C}$ across capacitor.

$t = 0$: Switch closes! Current starts to flow across resistor

$t = 0$



$$I(0) = \frac{V_0}{R} = \frac{Q_0}{RC}$$

$t \to \infty$ ... $I \to 0$, $Q \to 0$, $V \to 0$

what happens between $t=0$ and $t=\infty$?

$$I = -\frac{dQ}{dt} \qquad \underline{\text{sign}}: \quad Q\downarrow \; I\uparrow$$

$$= \frac{V}{R} = \frac{Q}{RC}$$

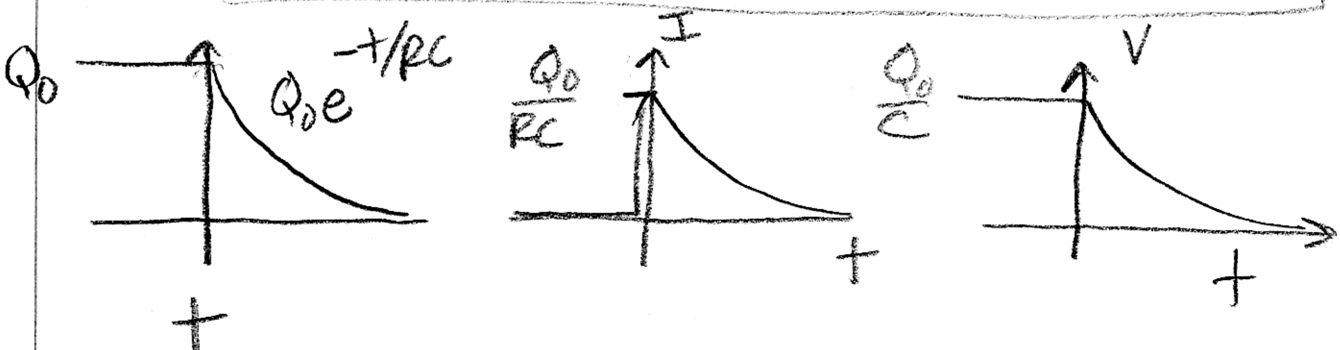
$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

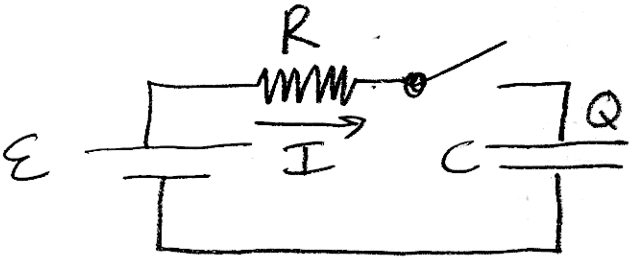
$$\ln Q = -\frac{t}{RC} + \text{constant}$$

$$Q = (\text{another Constant}) \cdot e^{-t/RC}$$

$t = 0, \quad Q = Q_0$

$$\boxed{\begin{aligned} Q &= Q_0 e^{-t/RC} \\ I &= -\frac{dQ}{dt} = \frac{Q_0}{RC} e^{-t/RC} \quad (t \geq 0) \\ V &= \frac{Q}{C} = RI = \frac{Q_0}{C} e^{-t/RC} \end{aligned}}$$





$Q=0,\ t<0$

Close switch at $t=0$.. what happens?

→ gradually C "charges up," gets more voltage..

→ $t=0$: current across resistor will be..

$$I = \frac{\mathcal{E}}{R}$$

→ all time

now: $\frac{dQ}{dt} = I$ (current charges, not discharges)

$$\mathcal{E} - R\frac{dQ}{dt} - \frac{Q}{C} = 0$$

$$\frac{1}{C}(Q - C\mathcal{E}) = -R\frac{dQ}{dt}$$

$$\tilde{Q} = Q - C\mathcal{E}$$

$$\frac{d\tilde{Q}}{dt} = \frac{dQ}{dt}$$

so $$\frac{d\tilde{Q}}{dt} = -\frac{\tilde{Q}}{RC} \Rightarrow \tilde{Q} = \tilde{Q}_0 e^{-t/RC}$$

$$Q = \hat{Q} + C\mathcal{E}$$

$$Q = \tilde{Q}_0 e^{-t/RC} + C\mathcal{E}$$

$$Q(0) = 0 = \tilde{Q}_0 + C\mathcal{E} \Rightarrow C\mathcal{E} = -\hat{Q}_0$$

$$\boxed{\begin{aligned} Q &= \left(1 - e^{-t/RC}\right) C\mathcal{E} \\ V &= \frac{Q}{C} = \left(1 - e^{-t/RC}\right)\mathcal{E} \\ I &= \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC} \end{aligned}}$$

$t > 0$