\[ \mathbf{J} = -e N_e \mathbf{v} \]

increases as one goes across.

decrease to compensate, when \( \frac{\partial N_e}{\partial x} = 0 \).

**Ohm's Law**

Empirically, in many substances,
\[ \mathbf{J} \propto \mathbf{E}, \quad \mathbf{J} = \sigma \mathbf{E} = \frac{1}{\rho} \mathbf{E} \]

\( \sigma \) a constant known as conductivity, \( \rho \) resistivity.

The larger \( \sigma \), the more current density for a given \( \mathbf{E} \) applied.

\[ \mathbf{J} = \sigma \mathbf{E} \]

leads to
\[ \mathbf{V} = \mathbf{I} \mathbf{R}, \]

```
V = IR,  "extrinsic"
```

In material
\[ \mathbf{E} = \frac{\mathbf{V}}{\ell}, \quad \mathbf{J} = \sigma \mathbf{E} = \frac{\sigma \mathbf{V}}{\ell} \]
\[ I = J \cdot A = \frac{\sigma VA}{L} \]

\[ V = \left( \frac{L}{\sigma A} \right) \cdot I \]

\[ \text{\(P\) (new quantity) = \frac{1}{\sigma} (\text{not charge density})} \]

\[ R = \frac{L}{A} \cdot \frac{1}{\sigma} = \frac{L}{A} \cdot \frac{1}{\sigma} \]

**Units:**

- CGS: $\frac{\text{esu}}{\text{cm}^2 \cdot \text{s}}$, $\frac{\text{esu}}{\text{cm}^2}$
- SI/MKS: $\frac{1}{\text{s}}$, $\frac{\text{esu}}{\text{cm}^2}$

There are some conditions for ohm's law... see text
### TABLE 4.1

Resistivity and its reciprocal, conductivity, for a few materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Resistivity $\rho$</th>
<th>Conductivity $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure copper, 273 K</td>
<td>$1.56 \times 10^{-6}$ ohm-cm</td>
<td>$6.4 \times 10^5$ (ohm-cm)$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$1.73 \times 10^{-18}$ sec</td>
<td>$5.8 \times 10^{17}$ sec$^{-1}$</td>
</tr>
<tr>
<td>Pure copper, 373 K</td>
<td>$2.24 \times 10^{-6}$ ohm-cm</td>
<td>$4.5 \times 10^5$ (ohm-cm)$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$2.47 \times 10^{-18}$ sec</td>
<td>$4.0 \times 10^{17}$ sec$^{-1}$</td>
</tr>
<tr>
<td>Pure germanium, 273 K</td>
<td>200 ohm-cm</td>
<td>0.005 (ohm-cm)$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$2.2 \times 10^{-10}$ sec</td>
<td>$4.5 \times 10^9$ sec$^{-1}$</td>
</tr>
<tr>
<td>Pure germanium, 500 K</td>
<td>0.12 ohm-cm</td>
<td>8.3 (ohm-cm)$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$1.3 \times 10^{-13}$ sec</td>
<td>$7.7 \times 10^{12}$ sec$^{-1}$</td>
</tr>
<tr>
<td>Pure water, 291 K</td>
<td>$2.5 \times 10^7$ ohm-cm</td>
<td>$4.0 \times 10^{-8}$ (ohm-cm)$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$2.8 \times 10^{-5}$ sec</td>
<td>$3.6 \times 10^4$ sec$^{-1}$</td>
</tr>
<tr>
<td>Seawater (varies with salinity)</td>
<td>25 ohm-cm</td>
<td>0.04 (ohm-cm)$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$2.8 \times 10^{-11}$ sec</td>
<td>$3.6 \times 10^{10}$ sec$^{-1}$</td>
</tr>
</tbody>
</table>

*Note: 1 ohm-meter = 100 ohm-cm = $1.11 \times 10^{-10}$ sec.*
Potential difference $V$

Conductivity $\sigma$

Current $I$

Length $L$

Area $A$

Current density $J = \frac{I}{A}$

Electric field $E = \frac{V}{L}$

Resistance $R = \frac{V}{I} = \frac{L}{\sigma A}$
One interesting example:

What is the same? \( \nabla \cdot \vec{J} \) when steady state

\[
\nabla \cdot \vec{J} = \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = \frac{\partial J_y}{\partial y} = 0
\]

in steady state.

What is different? \( \vec{E}_1 = \left( \frac{1}{\sigma_1} \right) \vec{J} < \left( \frac{1}{\sigma_2} \right) \vec{J} \)

(new field lines must start)

Must be some positive charge at the interface... in steady state, initially not (then, \( \frac{\partial \phi}{\partial x} \neq 0 \)) but it builds up.
Interesting Table - p. 133

\[ p = \frac{1}{\sigma} \]

Pure Copper: \( 1.56 \times 10^{-6} \) ohm-cm

\[ \sigma = 1.56 \times 10^{-6} \text{ S} \]

\[ 1 \text{ cm} \]

Pore Water: \( 25, 10^6 \) ohm-cm

Sea Water: \( 25 \) ohm-cm

(Bath Water)

Purity matters a lot!

Conductivity/Resistivity are very sensitive to temperature... look at plot on page 140.

Reason: dominant motion in condensed matter is thermal motion, even when electric field is present. Electric field is a minor effect.

Usually.
all moving (won’t draw all)

ion, $q_i$, $m_i$, $Mq_i$

NO ELECTRIC FIELD

$\Delta \vec{p} = q_i \vec{E} \Delta t$

$M \Delta \vec{v} = q_i \vec{E} \Delta t$

$\Delta \vec{v} = \frac{q_i \vec{E}}{M} \Delta t$

$\langle \Delta \vec{v} \rangle = \frac{q_i \vec{E}}{M} \langle \Delta t \rangle$

$\langle \vec{v} \rangle = \langle \Delta \vec{v} \rangle$

$\langle \vec{v} \rangle = \frac{q_i \vec{E}}{M} \langle \Delta t \rangle$

bunch of neutral atoms in solid, liquid

$\langle \vec{v} \rangle = 0$

$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} kT$

$\langle v^2 \rangle = \frac{3kT}{m}$

$\langle v^2 \rangle^{\frac{1}{2}} = \sqrt{\frac{3kT}{M}}$

Suppose: $T = 300 \text{ K}$, $m = 0.030 \text{ kg/mol}$

$\langle v^2 \rangle^{\frac{1}{2}} = \sqrt{\frac{3 \cdot 8.3 \cdot 300}{0.030}} = 500 \text{ m/s}$