Electric Current

is \frac{\text{charge}}{\text{second}} \rightarrow \text{flow of charge}

thin conducting wire \downarrow

\text{current} \xrightarrow{\text{flowing}} \text{net charge going through surface} \xrightarrow{\text{per unit time}} \text{is current}

CGS: units of current are \frac{\text{esu}}{\text{sec}}

MKS/SI: \frac{\text{coulombs}}{\text{second}} = \text{Ampere}

"Net Charge"

1 \text{esu In 1 second}

left to right

right to left

-1 \text{esu In 1 second}

SAME CURRENT AS

1 \text{esu In 1 second}

left to right

+1 \text{esu}

left to right

\text{net current is zero}

1 \text{esu/sec}

\text{some left} \quad \text{1 \text{esu/sec}} \quad \text{some move right}
Conclude: what matters, somehow, is the product of $q \vec{v}$.

Whoops check units: esu $\times \frac{\text{cm}}{\text{s}}$

"Back Up" to more fundamental picture.

- Area $\perp$ to direction $\vec{a}$

Herd of charges $N = \frac{\#}{\text{Volume}}$
Each has charge $q$

$I = \frac{(\text{Charge through area})}{\text{time}} = \frac{(nq \cdot a \cdot \text{velocity})}{\Delta t}$

$I = nq \cdot a \cdot \Delta t$
Now consider angle between $\vec{a} + \vec{v}$

cross section smaller, or, equivalently, volume smaller...

$I = n q_0 \cdot \vec{a}$

Usually, a variety of charges ($q_i$)
with a variety of densities ($n_i$) and
a variety of velocities ($\vec{v}_i$), (no variation
across $\vec{a}$)

Then $I = \left( \sum n_i q_i \cdot \vec{v}_i \right) \cdot \vec{a}$

why not name this something?

**Current Density** $\vec{J} = \sum n_i q_i \vec{v}_i$

$I = \vec{J} \cdot \frac{\vec{a}}{a} \leftrightarrow$ when $\vec{J}$ is constant across $\vec{a}$

when $q_i = q$, same (one species!) $\vec{J} = q \sum n_i \vec{v}_i = N_q q \left( \sum \frac{n_i \vec{v}_i}{N_q} \right)$

$N_q = \# q's, \text{ all velocities} \ $
average velocity \[ \vec{\mathbf{v}} = \frac{\sum_{i=1}^{N} \mathbf{v}_i}{N} \]
can be zero or small even when each \( \mathbf{v}_i \) is large, since many directions.

\[ \vec{J} = N_q \vec{q} \vec{v} \]

most common case: \( q = -e \) (electrons)
\[ \vec{J} = -Ne \vec{v} e \]

Current Conservation

more generally,
\[ I = \int_S \vec{J} \cdot d\vec{a} \]

For a closed surface \( S \)
\[ \int_S \vec{J} \cdot d\vec{a} = I = -\frac{dQ}{dt} = \frac{d}{dt} \int_V p \, dV \]

\( \uparrow \) surface around volume \( V \)

\( \uparrow \) current out means loss of charge
\[ \oint_{\partial V} \mathbf{J} \cdot d\mathbf{a} = \int_{V} \nabla \cdot \mathbf{J} \, dV = \int_{V} (-\frac{\partial \Phi}{\partial t}) \, dV \]

A inside is \[ \int_{\partial V} \mathbf{J} \cdot d\mathbf{a} \]

\[ \int_{V} \nabla \cdot \mathbf{J} \, dV = \int_{V} (-\frac{\partial \Phi}{\partial t}) \, dV \]

\[ \nabla \cdot \mathbf{J} = -\frac{\partial \Phi}{\partial t} \]

Steady state:

\[ \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} = -\frac{\partial \Phi}{\partial x} \]

Steady state:

\[ \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} + \frac{\partial J_z}{\partial z} \]

Current Conservation

Vacuum

Diode

Steady state.

\[ \frac{\partial J_x}{\partial x} = 0 \quad \text{(steady)} \]

\[ \text{density of electrons varies.} \]

\[ e^{-} \quad V_{2} \quad V_{3} \]

\[ \frac{\partial J_x}{\partial x} = 0 \quad \text{(steady)} \]

\[ \text{density of electrons varies.} \]

\[ e^{-} \quad V_{2} \quad V_{3} \]

\[ \text{acceleration} \]

\[ + \quad \text{voltage (anode)} \]

\[ - \quad \text{voltage (cathode)} \]

Neigher