when \( i > 0 \), \( \frac{1}{i} + \frac{1}{f} = \frac{1}{f} \), always greater than \( f \).

when \( i < 0 \), always \( > 0 \) for physical objects, but not otherwise.

when \( i < 0 \), \( \frac{1}{i} = -\frac{1}{f} - \frac{1}{o} < 0 \) when \( o < f \).

\[ f = \frac{1}{2} o \]

**Visual Technique**:

1. **Parallel In**

2. **Parallel Out**

3. **Center of curvature**

4. **Minor vertex**

\[ \text{all should agree} \]
Demo: multiple images (# bounces)

Total Internal Reflection

\[ n_2 \sin \theta_2 = n_1 \sin \theta_1 \leq \cos \theta_1 = 90^\circ \]

\[ n_2 \sin \theta_2 c = n_1 \]

\[ \sin \theta_2 c = \frac{n_1}{n_2} < 1 \]

\[ \theta_2 c = \sin^{-1} \left( \frac{n_1}{n_2} \right) = \sin^{-1} \left( \frac{1}{1.33} \right) \]

\[ = 48.8^\circ \text{ (from vertical)} \]

100% reflection! Better than mirror. Basis of fiber optics

\[ \sin \theta_c = \frac{1.52}{1.62} \quad \theta_c = 69.8^\circ \]
\[ T_{BA} = \frac{\sqrt{n_1^2 + x^2}}{(c/n_1)} + \frac{\sqrt{n_2^2 + (d-x)^2}}{(c/n_2)} \]

\[ \frac{dT}{dx} = \frac{n_1}{c} \frac{x}{\sqrt{n_1^2 + x^2}} - \frac{n_2}{c} \frac{(d-x)}{\sqrt{n_2^2 + (d-x)^2}} = 0 \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

This is an illustration of "Fermat's Principle," that...

\[ \int n(x) \, dx \] is an extremum path for actual path

Lenses
\[ \theta_1, \theta_2, \theta_r \]
\[ n_1, n_2, n_2 > n_1 \]

\[ \frac{\theta_1}{\theta_2} \]

\[ \text{small} \]
\[ \sin \theta_1 \sim \theta_1 \]
\[ \tan \theta_1 \sim \theta_1 \]
\[ \cos \theta_1 = 1 \]

\[ \theta' = \theta_1 + \theta_2 + \theta_r \]
\[ \theta_1 = \frac{h}{l} \]
\[ \theta_2 \sim \frac{h}{l} \]

\[ \theta_r \]
Lens Maker's Equation (thin lens)

\[ \frac{1}{f} = (n - 1) \frac{n}{r_i - r_2} > 0 \]

Virtual

Real

\[ - \frac{1}{r_i} < 0 \quad \frac{1}{r_2} > 0 \]

\[ f \] no matter what.

Thin lens: thickness irrelevant.

\[ i > 0 \]

\[ m = \frac{i}{o} \]

Converging real

3 principal rays:
1) First focus, out
2) In, through second
3) Undeflected through center

IGNORE: headed toward other
Diverging Lens

(no sign flip for real objects)

Fun with converging lenses

$0 < f$ : $i < 0$

\[
\frac{h'}{n} = \frac{-i}{0}
\]

\[
= \frac{f}{f-o}
\]

\[
\frac{1}{i} = \frac{1}{f} - \frac{1}{o} < 0
\]

\[
i = \frac{0-f}{f-o}
\]

magnifying glass

Best: $0 = f$
Lens System

\[ \frac{1}{f} + \frac{1}{\infty} = \frac{1}{f} \]

\[ -\frac{1}{f-L} + \frac{1}{i} = -\frac{1}{f} \]

\[ \frac{1}{i} = \frac{1}{f} + \frac{1}{f-L} = \frac{(f-L) + f}{f(f-L)} \]

\[ i = \frac{f}{f+L} \]

\[ i > 0 \]
When $\frac{1}{0} + \frac{1}{f} = \frac{1}{f}$, always than $f$

0: always $> 0$ for physical object, but

when $x < 0$

$\frac{1}{f} = \frac{1}{f} - \frac{1}{0} < 0$ when $0 < f$

Resid

Viside

\[ f = \frac{1}{2} \]

**Visual Technique:**

4 rays: 1) Parallel In $\uparrow$ focus

2) Through focus in $\Rightarrow$ Parallel Out

3) Center of curvature ... in/out

4) Minor vertex

1) all should agree
Total Internal Reflection

Air $n_1 \sim 1$

Water $n_2 \sim 1.33$

$n_2 \sin \theta_2 = n_1 \sin \theta_1$ \quad ($\max, \theta_1 = 90^\circ$)

$n_2 \sin \theta_2 < n_1$

$\sin \theta_2 < \frac{n_1}{n_2} < 1$

$\theta_{2c} = \sin^{-1} \left( \frac{n_1}{n_2} \right) = \sin^{-1} \left( \frac{1}{1.33} \right) = 48.8^\circ$ (from vertical)

100% reflection! Better than mirror! Basis of fiber optics

$\sin \theta_c = \frac{1.52}{1.62}$

$\theta_c = 69.8^\circ$
\[ T_{SA} = \frac{\sqrt{h_1^2 + x^2}}{c/n_1} + \frac{\sqrt{h_2^2 + (d-x)^2}}{c/n_2} \]

\[ \frac{dT}{dx} = \frac{n_1}{c} \frac{x}{\sqrt{h_1^2 + x^2}} - \frac{n_2}{c} \frac{(d-x)}{\sqrt{h_2^2 + (d-x)^2}} = 0 \]

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

This is an illustration of "Fermat's Principle," that:

\[ \int n(x) dx \] is an extremum for actual path

Lenses

\[ \theta_1 < \theta_r \]

\[ n_1 \theta_1 + \theta_2 + \theta_r \]

\[ n_2 > n_1 \]

\[ \theta_1 \text{ (very small)} \]

\[ \sin \theta_1 \sim \theta_1 \]

\[ \tan \theta_1 \sim \theta_1 \]

\[ \cos \theta_1 = 1 \]

\[ \theta_1' = \theta_1 + \theta_2 + \theta_r \]

\[ \theta_1 \sim \frac{h}{l_1} \quad \theta_2 \sim \frac{h}{l_2} \]
Lens Maker's Equation (thin lens)

\[
\frac{n}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2}) > 0
\]

Virtual

\[
\frac{n}{f} = \frac{1}{r_2} - \frac{1}{r_1} > 0 \quad f < 0 \quad \text{no matter what.}
\]

Thin lens: thickness irrelevant.

\[
m = \frac{i}{o} \quad \text{(magnification)}
\]

3 principal rays:
1) First focus, out II
2) In II, through second
3) Undeflected through center

IGNORE: headed toward other
Diverging Lens

For real objects
(no sign flip
for converging lenses)

Fun with converging lenses

\[
\frac{1}{o} = \frac{1}{i} + \frac{1}{h'} \quad o < 0
\]

\[
h' = \frac{hi}{o - i}
\]

Post: \( o = + \)

\( \text{Negative glass} \)
**Lens System**

\[
f + \frac{1}{i} + \frac{1}{o} = \frac{1}{f}
\]

Virtual object, converging.

\[
\frac{1}{o'} + \frac{1}{i} = -\frac{1}{f}
\]

\[
\frac{1}{f-L} + \frac{1}{i} = -\frac{1}{f}
\]

\[
\frac{1}{i} = -\frac{1}{f} + \frac{1}{f-L} = \frac{-L}{f(L-f)} + \frac{f}{f(f-L)}
\]

\[
i = \frac{f}{L} \frac{f(L-f)}{>0}
\]

NET CONVERGENCE
Magnifying Glass

Really helps your near point

Object → $P_n$ smallest still in focus

$\theta_{\text{max}} = \frac{h}{P_n}$

$e = \frac{i}{f - o}$

$\theta - h' = \frac{-i}{i_0}$

$\theta' = \frac{P_n}{f}$

$f < P_n$

Slight undeflected ray

Object: eye can handle

$nearly parallel$
Microscope

Add a stage to "blow up" little image.

\[ h \sim \frac{h}{f_{obj}} \]

\[ \theta \sim \frac{h}{P_n} \]

\[ h \sim P_n \theta \]

\[ \frac{h'}{s} = \frac{h}{f_{obj}} \]

\[ h' = \frac{h}{s} \]

\[ \frac{\theta'}{f_{eye}} = -\frac{h s}{f_{obj} f_{eye}} \sim -\frac{P_n s}{f_{obj} f_{eye}} \]

\[ M = \frac{\theta'}{\theta} = \frac{P_n s}{f_{obj} f_{ex}} \]
Telescope

Like microscope, but shift of interest \( \infty \) far away

\[ h' = -\frac{f_{obj}}{\theta} \]

\[ \frac{\theta'}{\theta} = \frac{h'}{f_{eye}} \]

Parallel rays in

Key concept: off axis parallel rays converge in focal plane