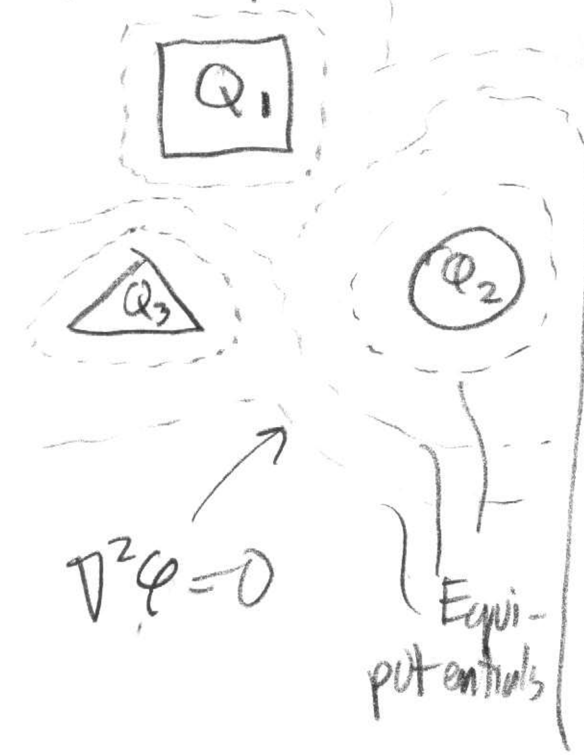


"Boundary Condition"

1. shapes of the conductors
2. (a) Potenials of them (or some of them)
   (b) Charge on them (or some of them)
3. in between them $\nabla^2\varphi = 0$
4. $\phi \to 0$ as $r \to \infty$

How many solutions?

(A) ~~none~~ ⇐ physics

(B) exactly one ⇐ That is it.

(C) more than one

"<u>Uniqueness</u>": $\varphi(x,y,z)$

<u>suppose</u> two solutions $\psi(x,y,z)$

Both Satisfy boundary conditions... at the conductors and

$\nabla^2\varphi = 0$, $\nabla^2\psi = 0$ outside

Consider

$$W(x,y,z) = \varphi(x,y,z) - \psi(x,y,z)$$

(1) At the conductors, $W(\underbrace{x_c, y_c, z_c}_{\text{points on conductors}}) = 0$

because

$$\varphi(x_c, y_c, z_c) = \psi(x_c, y_c, z_c)$$

(same boundary conditions)

(2) $\nabla^2 W =$

$$\frac{\partial^2}{\partial x^2}\left(\varphi(x,y,z) - \psi(x,y,z)\right)$$

$$+ \frac{\partial^2}{\partial y^2}\left(\varphi(x,y,z) - \psi(x,y,z)\right)$$

$$+ \frac{\partial^2}{\partial z^2}\left(\varphi(x,y,z) - \psi(x,y,z)\right)$$

$$= \frac{\partial^2}{\partial x^2}\varphi + \frac{\partial^2}{\partial y^2}\varphi + \frac{\partial^2}{\partial z^2}\varphi$$

$$+ \frac{\partial^2}{\partial x^2}\psi + \frac{\partial^2}{\partial y^2}\psi + \frac{\partial^2}{\partial z^2}\psi$$

$$= \nabla^2\varphi + \nabla^2\psi = 0$$

③ $W = 0$ everywhere ...

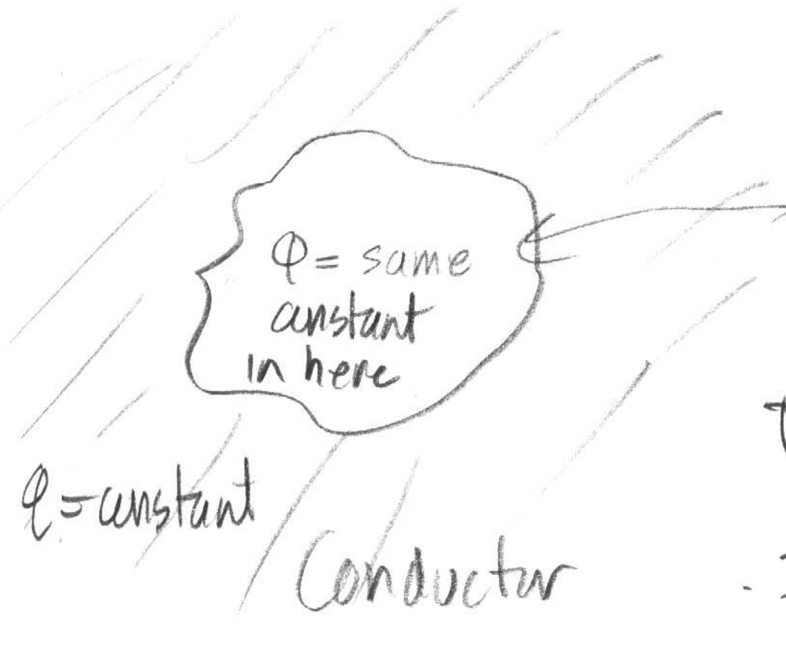
$W = 0$ on conductors

$W = 0$ as $r \to \infty$

if $W \neq 0$ everywhere, must be a maximum (or minimum)... cannot be true, by averaging property

④ $\varphi(x,y,z) = \psi(x,y,z)$

<u>Consequence</u>: Cavity



matches boundary conditions

$\nabla^2 \varphi = 0$

∴ it is the solution

$\vec{E} = 0$ inside then, since

$-\vec{\nabla}\phi = 0$