Conducting bar in uniform $\vec{B}$ field

$\vec{B}$ everywhere $\vec{E} = \vec{0}$

$q \cdot \vec{v} \times \vec{B}

\vec{F} = q \frac{\vec{v}}{c} \times \vec{B}$

$\vec{F} = q \vec{v} B \times \frac{1}{c}$

$\vec{E} = \frac{\vec{v} B}{c}$

What happens now? +++++ on one end (+x)

--- on other (-x)

Ends of rods charge up until current flow stops...

How much? Depends on $R$, $C$ of time rod!

Look at Figure 7.3 --- key points...

- Direction $\vec{E}$ in a conductor $\ldots$ $\vec{E} = 0$, not $\vec{E}$

- Density of field lines
7.3 shows some electric field lines in the frame $F'$, and in the magnified view of the end of the rod in Fig. 7.4b we observe that the electric field inside the rod is zero.

Except for the Lorentz contraction, which is second order in $v/c$, the charge distribution seen at one instant in frame $F$, Fig. 7.3b, is the same as that seen in $F'$. The electric fields differ because the field in Fig. 7.3 is that of the surface charge distribution alone, while the electric field we see in Fig. 7.4 is the field of the surface charge distribution plus the uniform electric field that exists in that frame of reference. An observer in $F$ says: “Inside the rod there has developed an electric field $E = (v/c) \times B$, exerting a force $qE = -q(v/c) \times B$ which just balances the force $q(v/c) \times B$ that would otherwise
"Inside the rod there is no electric field, and although there is a uniform magnetic field here, no force arises from it because no charges are moving." Each account is correct.
What happens in bar's rest frame?
\[ \vec{V}' = 0 \] so no magnetic force
\[ \vec{q} \times \vec{E}' = \vec{0} \]
\[ \vec{E}' = \vec{0} \]

How?
\[ \vec{E}'_L = \gamma (\vec{E}'_L + \vec{\beta} \times \vec{B}'_L) \]
\[ \frac{-\gamma}{c} \vec{B} \times \vec{\beta} \]

\[ \frac{-\gamma}{c} \vec{B} \times \vec{\beta} = \vec{0} \]

Figure 7.4

\[
\begin{array}{c}
\text{LOOP!} \quad \text{at rest...} \\
of \text{conductor} \quad \vec{\beta} \\
\frac{\gamma}{c} \vec{B} \\
\vec{F} \\
\vec{\beta} \\
\vec{B} \\
\text{moving in...} \\
\text{UNIFORM} \\
\text{LOOP} \\
S \vec{F} \cdot d\vec{s} \\
= 0! \\
\text{BUT NOW TWO} \quad \text{it's cancel!} \\
\end{array}
\]
\[ \oint \vec{F} \cdot d\vec{s} = \frac{q}{c} V (B_1 - B_2) w \]

**Loop**

\[ \varepsilon = \frac{1}{q} \oint \vec{F} \cdot d\vec{s} \quad \text{"electro motive force"} \]

In this frame, \[ \oint \vec{E} \cdot d\vec{s} = 0 \]

but... \[ \oint \vec{E} \cdot d\vec{s} \quad \text{Loop rest frame...} \]

Makes current flow in loop.

I: What is \( I \) trying to do?

\[ \Rightarrow \text{INCREASE } \frac{B}{B_0} \] as \( B \) as seen by the loop decreasing.
LENZ's LAW: Induced currents try to re-establish \( \mathbf{B} \)

"Ampere" like relationship

"Penetrating" \( \mathbf{B} \) \( \mathbf{J} \)

Faraday's Law of induction

\[ \oint \mathbf{B} \cdot d\mathbf{A} = \frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A} \]

\( \mathbf{B} \) change

\( I \neq 0 \) role: induced current makes a new field that opposes the change.
How do we quantify this? (142)

\[ E = \frac{VW}{c} (B_1 - B_2) \]

In general...

\[ E = -\frac{1}{c} \frac{d\Phi}{dt} \]

1. ID the loop

2. surface

3. Compute \( \int B \cdot d\mathbf{a} = \Phi \)
(4) Choice of surface... doesn't matter (Soap Bubble Videos).

(5) \( \frac{d\Phi}{dt} \) matters!

(6) - sign is... "Lenz Law"

Connect with loop.

\[
\frac{d\Phi}{dt} = (B_2 V dt - B_1 V dt) W
\]

\[
\frac{1}{C} \frac{d\Phi}{dt} = \frac{V}{C} W (B_2 - B_1)
\]

\[
\frac{1}{C} \frac{d\Phi}{dt} = \frac{V W (B_1 - B_2)}{C}
\]
Another Classic

Fig 7.13

\[ \Phi \text{ biggest when } \theta = 90^\circ \]

\[ \Phi = SB \sin(\omega t + \alpha) \]

\[ \alpha = 0, \text{ actually.} \]

\[ \frac{d\Phi}{dt} = \omega SB \cos(\omega t + \alpha) \]

\[ \Delta V = -\frac{1}{c} \frac{d\Phi}{dt} = -\frac{\omega SB}{c} \cos(\omega t + \alpha) \]

At what \( \theta \) is \( \Delta V \) biggest?

\( \theta = 0 \)

\( \theta = 90^\circ \)

\( \theta = 45^\circ \)?

Which way?
Is circuit moving the only way?

\[ I \neq ? \]

YES... different Lorentz frame...

In this frame, \( \int E' \cdot ds' = 0 \)

Third case:

Stationary. Still current flows.
Odd derivation... \( Q_v = I \ell \)

Wire

\( n = \text{# charges / volume} \)
\( q = \text{charge} \)
Area \( A \)

\[
\ell (\text{length}) \quad 1
\]

\[
Q = nq \cdot A \cdot \ell
\]

\[
Q_v = nq \cdot A \ell \nu
\]

\[
\frac{q}{(nv)} \cdot A \cdot \ell
\]

\[
\frac{1}{I}
\]

\[
Q_v = \frac{1}{I} \ell
\]
General Law of Induction:

\[ \oint \mathbf{E} \cdot d\mathbf{s} = \mathcal{E} = \frac{-1}{c \partial t} \int \mathbf{B} \cdot dr \]

\[ \frac{d\Phi}{dt} = -\frac{1}{c \partial t} \]

Stokes

\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \]

(maxwell)

Forces from Lenz

move toward...
ring should move away

creates \( \mathbf{B} \) that cancels...

need \( \frac{d\Phi}{dt} \) to get I in ring
Fig 7.16 of Purcell...

Current: \( I = I_0 \sin(\frac{377}{2\pi \cdot 60} \cdot t) \)

\[ f \text{ to } w \text{ 60 cycle} \]

\( B_{max} = 50 \text{ G}, B = B_{max} \times \sin(377t) \)

\( r = 10 \text{ cm} \)

\[ \Phi = \pi r^2 B \]

\[ = \pi (10)^2 50 \sin((377 + t)) \]

\[ \Phi = 15,700 \sin((377 + t)) \]

\[ E = -\frac{1}{c} \frac{d\Phi}{dt} \]

\[ = -\frac{1}{c} 15,700 \cdot 377 \cos((377 + t)) \]
The magnetic field \( B \) in the central region reaches a maximum value of 50 gauss. We want to investigate the induced electric field, and the electromotive force, on the circular path 10 cm in radius shown in Fig. 7.16. We may assume that the field \( B \) is practically uniform in the interior of this circle, at any instant of time.

\[
B = 50 \sin 377t
\]  
(32)

\( B \) is in gauss and \( t \) in sec. The flux through the loop \( C \) is

\[
\Phi = \pi r^2 B = \pi \times 10^2 \times 50 \sin 377t = 15,700 \sin 377t \quad \text{(gauss-cm}^2\text{)}
\]

(33)

Using Eq. 31" to calculate the electromotive force in volts,

\[
\mathcal{E} = -(10^{-8}) \frac{d\Phi}{dt} = -(10^{-8})(377)(15,700) \cos 377t
\]

\[
= -0.059 \cos 377t \quad \text{(volts)}
\]

(34)

The maximum attained by \( \mathcal{E} \) is 59 millivolts. The minus sign will ensure that Lenz' law is respected, if we have defined our directions consistently. The variation of both \( \Phi \) and \( \mathcal{E} \) with time is shown in Fig. 7.17.

What about the electric field itself? Usually we cannot deduce
FIGURE 7.17
(a) The flux through the circle C. (b) The electromotive force associated with the path C.

FIGURE 7.18
The electric field on the circular path C. (a) In the absence of sources other than the symmetrical, oscillating current. (b) Including the electrostatic field of two charges on the axis.

\[ \Phi = 15,700 \text{ gauss-cm}^2 \]

\[ \epsilon_{\text{max}} = 0.059 \text{ volt} \]

E from a knowledge of curl E alone. However, our path C is here a circle around the center of a symmetrical system. If there are no other electric fields around, we may assume that, on the circle C, E lies in that plane and has a constant magnitude. Then it is a trivial matter to predict its magnitude, since \( \int_C \mathbf{E} \cdot d\mathbf{s} = 2\pi rE = \epsilon \), which we have already calculated. In this case, the electric field on the circle might look like Fig. 7.18a at a particular instant. But if there are other field sources, it could look quite different. If there happened to be a positive and a negative charge located on the axis as shown in Fig. 7.18b, the electric field in the vicinity of the circle would be the superposition of the electrostatic field of the two charges and the induced electric field.

**MUTUAL INDUCTANCE**

7.6 Two circuits, or loops, \( C_1 \) and \( C_2 \) are fixed in position relative to one another (Fig. 7.19). By some means, such as a battery and a variable resistance, a controllable current \( i \) is caused to flow in circuit \( C_1 \). Let \( B_i(x, y, z) \) be the magnetic field that would exist if the current
in $C_1$ remained constant at the value $I_1$, and let $\Phi_{21}$ denote the flux of $B_1$ through the circuit $C_2$. Thus

$$\Phi_{21} = \int_{S_2} B_1 \cdot da_2$$  \hspace{1cm} (35)

where $S_2$ is a surface spanning the loop $C_2$. With the shape and relative position of the two circuits fixed, $\Phi_{21}$ will be proportional to $I_1$:

$$\frac{\Phi_{21}}{I_1} = \text{const}$$  \hspace{1cm} (36)

Suppose now that $I_1$ changes with time, but slowly enough so that the field $B_1$ at any point in the vicinity of $C_2$ and the current $I_1$ in $C_1$ at the same instant of time are related as they would be for steady currents. (To see why such a restriction is necessary, imagine that $C_1$ and $C_2$ are 10 meters apart and we cause the current in $C_1$ to double in value in 10 nanoseconds!) The flux $\Phi_{21}$ will change in proportion as $I_1$ changes. There will be an electromotive force induced in circuit $C_2$, of magnitude

$$\mathcal{E}_{21} = -\frac{\text{const}}{c} \frac{dI_1}{dt}$$  \hspace{1cm} (37)

The constant here is the same as the one in Eq. 36. Let's absorb the $c$ in the denominator into a single constant, denoted by $M_{21}$, and write Eq. 37 in this way:

$$\mathcal{E}_{21} = -M_{21} \frac{dI_1}{dt}$$  \hspace{1cm} (38)

We call the constant $M_{21}$ the coefficient of mutual inductance. Its value is determined by the geometry of our arrangement of loops.